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DETERMINING THE FLANKS OF THE BEVEL GEARS WITH ELOID TEETH USING MATHEMATICAL EQUATIONS

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Abstract: This paper presents the mathematical equations of the bevel gears' flanks with eloid teeth. These gears belong to the class of curved teeth which guide-line of tooth is a curled epycicloid curve with constant height of tooth. These equations base on the face gear's tooth flanks which are used as cutting tools. The mathematical equations are calculated with transformations of the coordinate systems. For these transformations homogenous matrixes are used. The obtained surfaces, which are point clouds, are determined in Matlab software.

1. INTRODUCTION

The gear's history dates back to the first civilizations. These gear's teeth were like stakes fixed on the wheel's circumference. This form was used until the astronomer De la Hire acknowledged the importance of the profile curve, [1]. Today the involute curve is used which was invented by the mathematician Leonhard Euler.

The generation of the bevel gears is realized by a cutting tool using one or two teeth of the face gear. This cutting tool has a straight trapezoidal profile, so the profile of the generated bevel gear is not spherical involute - it is called octoid teething, [4].

First there were used the bevel gears with straight teeth but as the transferred power grew, the bevel gear with oblique and curved teeth appeared. In practice, bevel gears with oblique teeth are rarely used.

Bevel gears with eloid teeth belong to the class of curved teeth. Its guide-line of tooth is a curled epycicloidal curve with constant height of the tooth.

The objective of this paper is to present the flanks generation of the bevel gears with eloid teeth using mathematical equations. These relations are determined using the generating face gear's teeth as a cutting tool. The surfaces which are point clouds are determined in Matlab software. It is possible to change the initial parameters (exterior frontal module, pressure angle, number of teeth, pitch angle, base helix angle, group of tools) to obtain other bevel gears with eloid teeth with other parameters.

2. THE ELOID FACE GEAR

The guide-line of tooth of the face gear with eloid teeth is an epycicloid curve described by an exterior point *P*, of the rolling circle with radius of r_r. This rolling circle rolls on the base circle r_b without slipping (Figure 1)

The eloid teeth generation is realized with head-cutter which radius is r_s, that rotates around its axes with the rolling circle's angular velocity. The combination of the rotary motion between the head-cutter and the face gear the teeth are generated. So, in each *P* point is mounted a cutting tool with straight trapezoidal profile. To simplify the generation process, there are located in every *P* point a group of 2..5 cutting tool. Ordinarily, there are 3 cutting tools in a group, where:

- the first performs the roughing of the gap between the teeth;
- the next two tools process the convex and concave flanks.

It is very important that the teeth generation is realized by continuous division.

The height of the tooth is constant for eloid teeth. Because of this, rolling takes place between the rolling plane of the generating face gear and the rolling cone of the bevel gear. This means that the eloid teeth is an octoid I teeth, [2, 4].

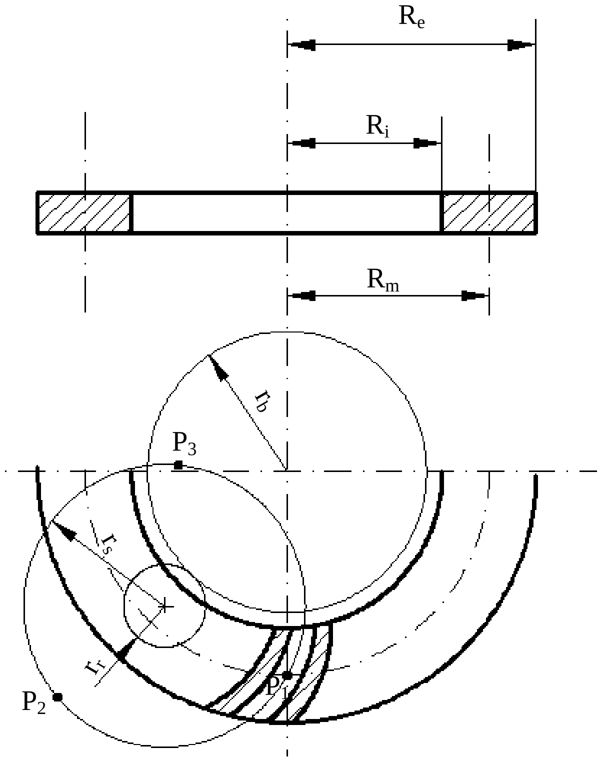

Figure 1. The eloid bevel gear generation, [4]

2.1.THE EQUATIONS OF THE GUIDE LINE OF TOOTH IN 2D

As the tools are mounted on the head-cutter with r_s radius, the initial equations in 2D are:

$$(r_s):\begin{cases} X_s = 0 \\ Y_s = r_s \end{cases} \tag{2.1}$$

In figure 2 are presented the relative movements between the imaginary face gear (the negative of the generating face gear) and the cutting tool.

The presented coordinate systems are:

- $O_s^0X_s^0Y_s^0Z_s^0$ – the initial coordinate system of the cutting tool head;
- $O_sX_sY_sZ_s$ – the mobile coordinate system of the cutting tool head;
- $O_aX_aY_aZ_a$ – auxiliary system;
- $O_1X_1Y_1Z_1$ – the mobile coordinate system of the imaginary face gear;
- $O_0X_0Y_0Z_0$ – the final coordinate system fixed with the imaginary face gear.

These coordinate systems define the movements during the generation. The motions can be determine using matrixes. Making their products the final matrix that contains the movements is:

– $$M_1 = \begin{bmatrix} \cos(\varphi_{1s}+\varphi_s) & \sin(\varphi_{1s}+\varphi_s) & (r_b+r_r)\cdot\cos(\varphi_{1s}+\theta) \\ -\sin(\varphi_{1s}+\varphi_s) & \cos(\varphi_{1s}+\varphi_s) & -(r_b+r_r)\cdot\sin(\varphi_{1s}+\theta) \\ 0 & 0 & 1 \end{bmatrix} \tag{2.2}$$

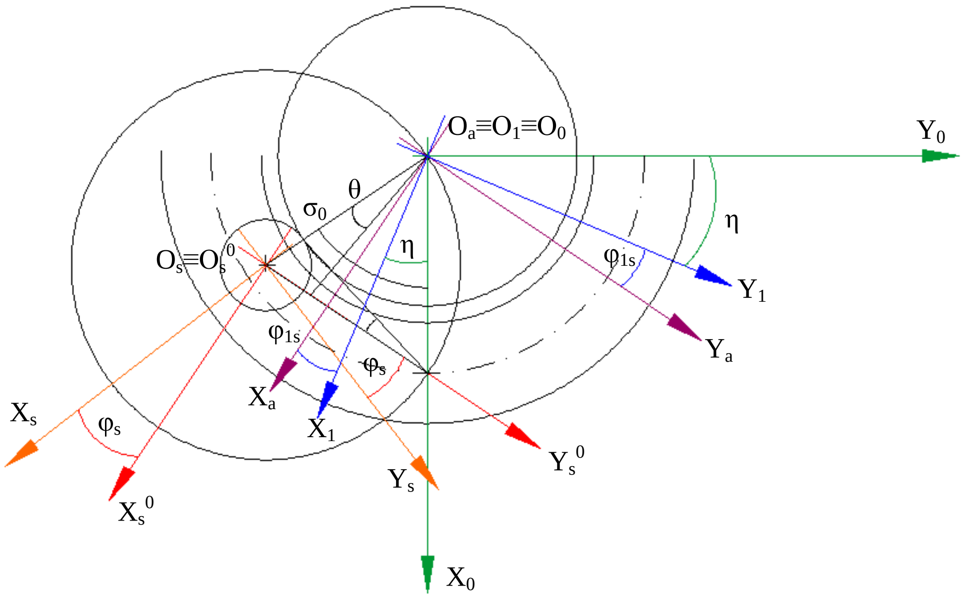

Figure 2. Representation of the coordinate systems to achieve the flanks of the face gear with eloid teeth

Substituting the equations (1) and (2) in (3) the guide-line of tooth is determined in the face gear mobile coordinate system:

$$r_1 = M_1 \cdot r_s \tag{2.3}$$

To obtain the guide-line of tooth in the $O_0X_0Y_0Z_0$ coordinate system an auxiliary rotation is required with the η angle.

The initial parameters are: m_e = 5.64 mm; α = 17.5^o; z_1 = 16; δ = 32.62^o; β = 38.22^o; z_S = 5.

The obtained equations have two parameters. There is a correlation between these, called equation of meshing [5, 6], that fulfils the equality of the angular velocity between the rolling circle and the base circle. That means the rolling circle rolls without slipping on the base circle. The obtained equations are expressed in φ_{1s} parameter.

2.2.THE CONVEX AND CONCAVE FLANKS OF THE FACE GEAR

The convex and concave flanks of the face gear base on determining the guide-line. The (1) equations may turn out as follows:

- the convex flank:

$$(\Sigma_{convex}): \begin{cases} X_s(u) = 0 \\ Y_s(u) = r_s - u \cdot tg\alpha \\ Z_s(u) = u \end{cases} \tag{2.4;}$$

- the concave flank is generated from the convex, which is rotated with a φ angle along the milling head axes:

$$(\Sigma_{concave}): \begin{cases} X_s(u) = (r_s + u \cdot tg\alpha)\cdot \sin\varphi \\ Y_s(u) = (r_s + u \cdot tg\alpha)\cdot \cos\varphi \\ Z_s(u) = u \end{cases} \tag{2.5.}$$

The u is a variable parameter that helps to determine coordinate points on the whole working depth of the tooth.

The resulting equations, (4) and (5), are substituted in equation (3) so the convex and concave flanks of the face gear are determined, presented in figure 3.

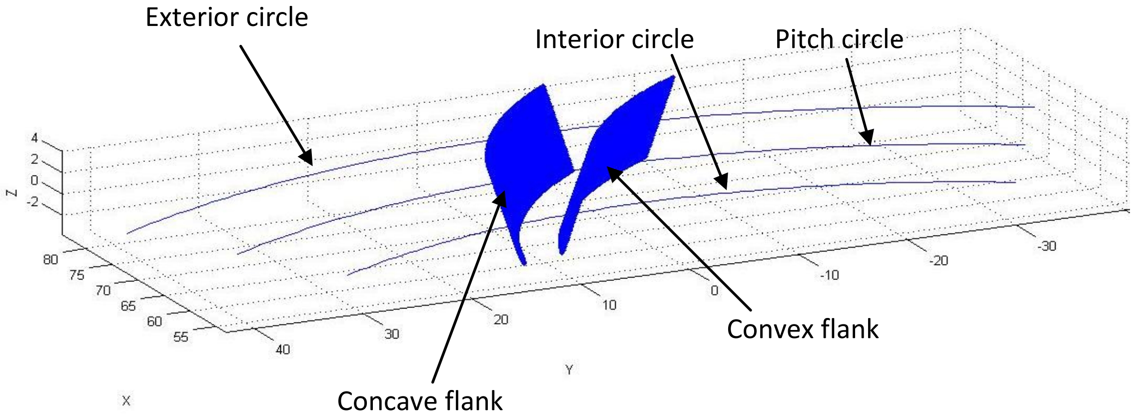

Figure 3. The convex and concave flanks of the face gear

3. THE ELOID BEVEL GEAR

The bevel gear tooth surface equations determination aims the idea of envelope surfaces, [7]. It starts from the flanks' surface equations of the face gear, obtained previously, and requires the establishment of the tooth flanks' equations of the face gear.

For this the equations of the face gear which are determined in its own coordinate system have to express in the bevel gear's coordinate system.

According to figure 4, three transformations are required which are identified as three homogenous matrixes products:

1. along the Z axis:

$$Z_1 \equiv Z_1^{(0)} \equiv Z_0; X_1 \equiv X_1^{(0)} \rightarrow X_0; Y_1 \equiv Y_1^{(0)} \rightarrow Y_0 \quad (3.1);$$

2. along the Y axis:

$$Y_0 \equiv Y_2^{(0)}; X_0 \rightarrow X_2^{(0)}; Z_0 \rightarrow Z_2^{(0)} \quad (3.2);$$

3. along the X axis:

$$X_2^{(0)} \equiv X_2; Y_2^{(0)} \rightarrow Y_2; Z_2^{(0)} \rightarrow Z_2 \quad (3.3).$$

The presented coordinate systems are:

– $O_0X_0Y_0Z_0$ the reference (world) coordinate system;
– $O_1X_1Y_1Z_1$ the mobile coordinate system of the face gear;
– $O_1^{(0)}X_1^{(0)}Y_1^{(0)}Z_1^{(0)}$ the fix coordinate system of the face gear;
– $O_2X_2Y_2Z_2$ the mobile coordinate system of the bevel gear;
– $O_2^{(0)}X_2^{(0)}Y_2^{(0)}Z_2^{(0)}$ the fix coordinate system of the bevel gear.

The φ_1 and φ_2 are rotation angles of the face gear and the bevel gear; δ is the pitch angle.

The resulting equations are expressed in the bevel gear's coordinate system and have four parameters: *u, φ1s, φ1, φ2*. To be realized the meshing between the face gear and the bevel gear the angular velocity of them has to be equal. Thus the equations become:

$$\begin{cases} X = X_2(u, \varphi_{1s}, \varphi_1) \\ Y = Y_2(u, \varphi_{1s}, \varphi_1) \\ Z = Z_2(u, \varphi_{1s}, \varphi_1) \end{cases} \quad (3.4).$$

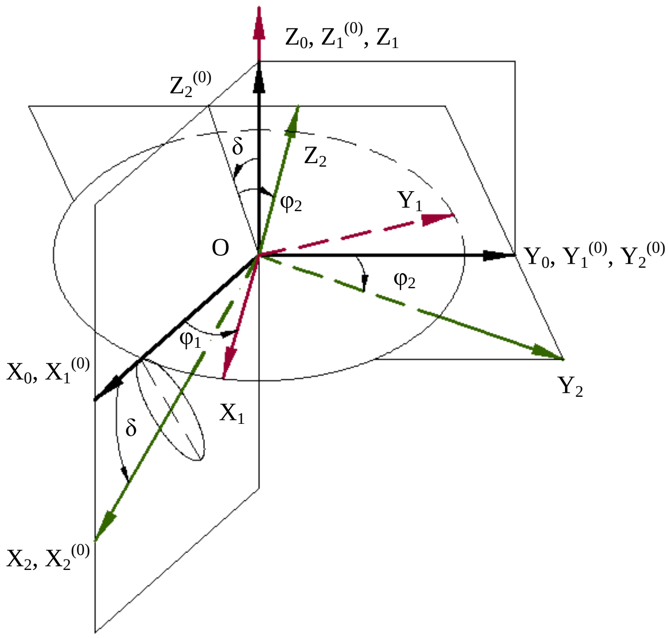

Figure 4. Coordinate transformations for bevel gear, [8]

To obtain the equations of the flanks' surfaces the correlation between these parameters are determined with the equation of meshing, [5, 6].

The equations (3.5) determine the flanks of the bevel gear with eloid teeth in its own coordinate system presented in figure 5.

$$\begin{cases} X = X_2(u, \varphi_{1s}) \\ Y = Y_2(u, \varphi_{1s}) \\ Z = Z_2(u, \varphi_{1s}) \end{cases} \qquad (3.5).$$

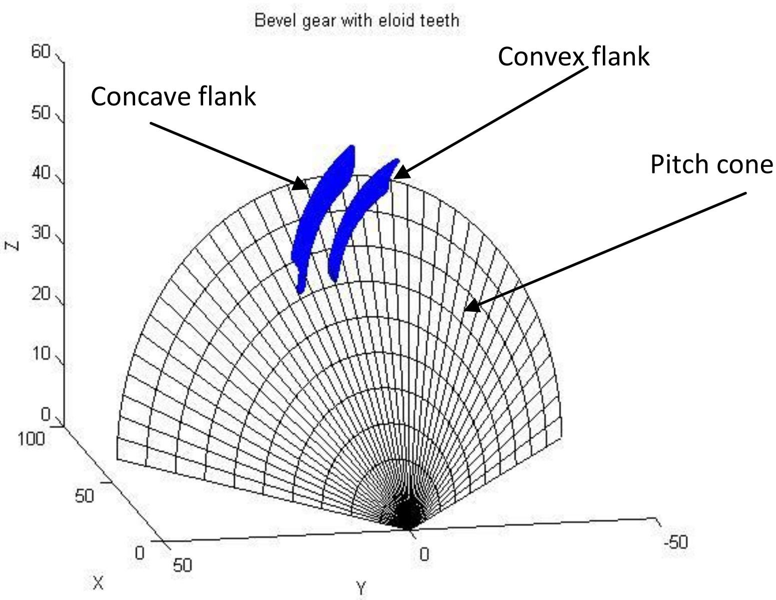

Figure 5. Bevel gear with eloid teething generated in Matlab software

4. CONCLUSIONS

The objective of this paper is to present the flanks generation of the bevel gears with eloid teeth using mathematical equations is achieved. The relations are determined using generating face gear’s teeth. The obtained surfaces, which are point clouds, are determined in Matlab software.

As further development can be a graphical user interface, presented a primitive version in [3]. Introducing the initial parameters coordinate points of the flanks’ surfaces are obtained. These coordinate points can be transported in CAD software to model the theoretical bevel gear.

The point of view of measuring, the real flanks surfaces of the eloid bevel gear can be compared with the surfaces obtained by mathematical equations.

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