ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume XI (XXI), 2012, NR2

SOME SPECIFICATIONS CONCERNING EULER'S ANGLES IN THE STUDY OF RIGIDIS'S MOVEMENT WITH A FIX POINT

Mihuţ Nicoleta - Maria
University CONSTANTIN BRÂNCUŞI of Târgu Jiu
nicoleta_simionescu@yahoo.com

Keywords: Euler's angles, precessional movement, nutation movement, own rotation movement

Abstract: As part of this paper it is accentuated the modality in which, the mobile system of axes, connected to the rigid in movement and initially situated superposed on the fix system of axes, in rapport with which can be studied the movement of the rigid, can be brought in a certain position, impressing successively three rotations around an axis passing by the fix point, with angles which are assumed to be precisely Euler's angles.

1. INTRODUCTION

There are considered (fig.1.a) the two systems of fix axes $Ox_1y_1z_1$ and initials Oxyz superposed. In fig.1.b these systems of axes, maintaining the initial origin $0_1=0$, are totally different. Let's observe the movement of the system of mobile axes from position 1.a in position 1.b.

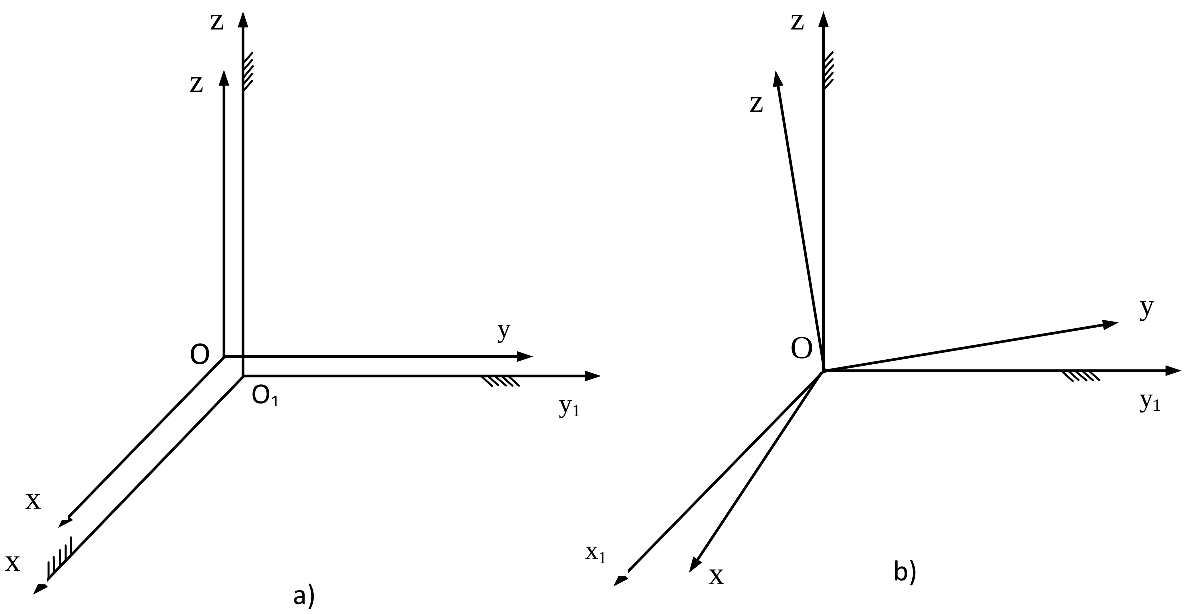

Figure 1.

It is given the mobile system of axes, a rotation around a fix axle Oz, name the precessional movement of angle ψ called precession angle, in a trigonometric sense, like in figure 2. Following this movement, the axes Oz and Oz_1 remain superposed in fix planes $Ox_1y_1z_1$ and mobile Oxyz and also superposed like in fig.2. The vector of the angular speed of the precessional motion would be in this case $\dot{\psi} k_1$ oriented on the fix axis Oz_1.

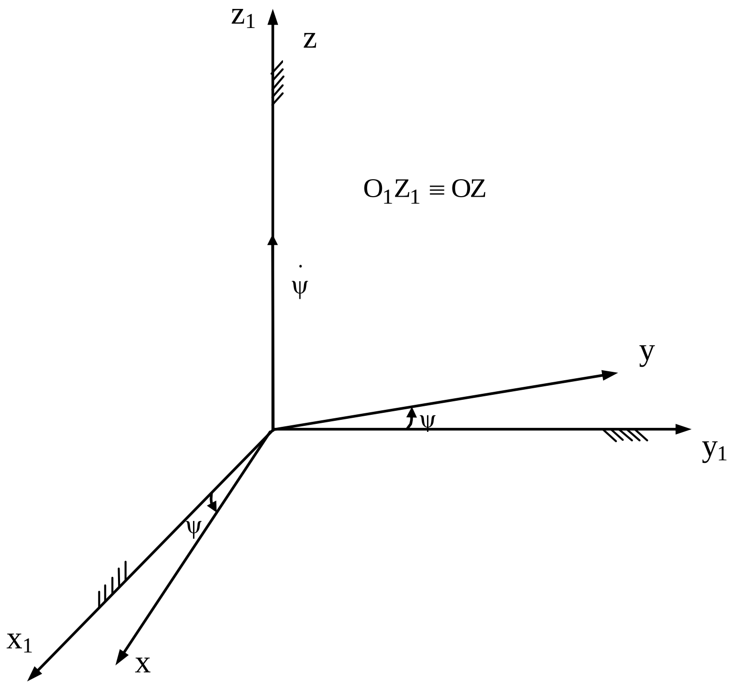

Figure 2. The precessional movement

Starting further from the position of the system of mobile axes in respect to the fix one given in fig.3. a rotation movement is impressed to the mobile system of axes, called nutation movement, as can be seen in fig.3. The angle θ is called nutation angle, and the rotation axis is called node's line ($O_1N \equiv OZ$). The speed of the angle corresponding to this rotation movement would be a vector oriented on the line of the nodes $\dot{\theta}$. The fix plan $Ox_1y_1z_1$ and the mobile one Oxyz are not anylonger superposed like in fig.2, but the axis Ox remains further perpendicular on Oz, which makes that the mobile system of axes is not yet in a totally different position from the fix one.

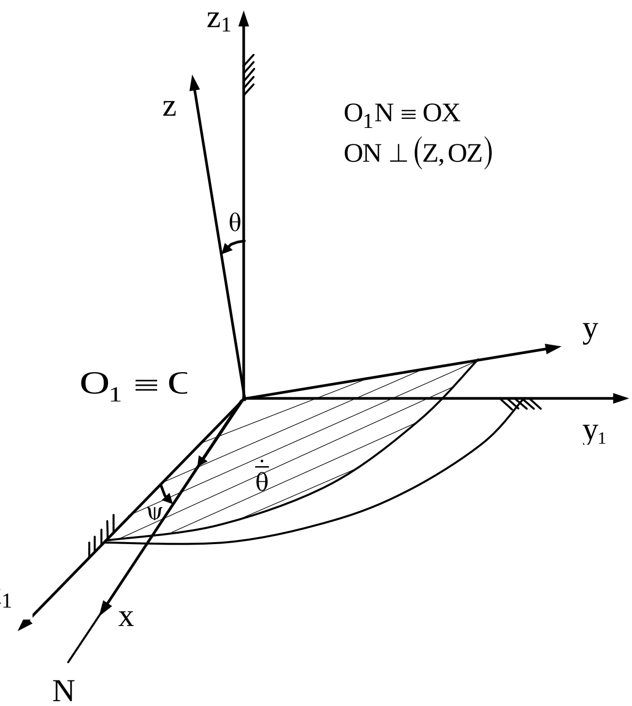

Figure 3. The nutation movement

In case afterwards the mobile system of axes rotates around own axis Oz like in fig.4. with a rotation angle φ in a trigonometric sense, called angle of own rotation, that movement being called <u>own rotation movement</u>, the two systems of mobile axes are in totally different positions, the axis Ox not being now perpendicular on Oz_1. The angular speed of this movement would be vector $\dot{\psi}$ oriented on the mobile axis Oz according to the right borer rule, like the other angular speeds of procession and rotation.

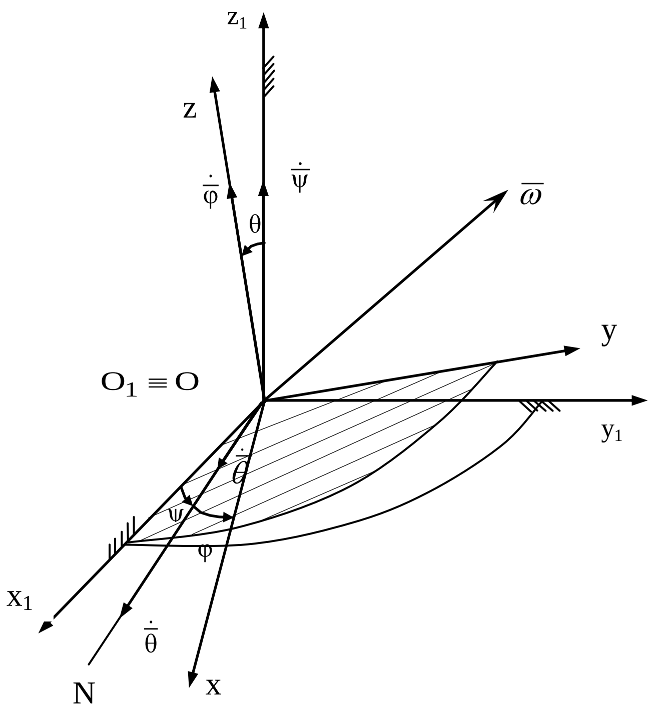

Figure 4. The own rotation movement

The three angular speeds $\dot{\psi}$, $\dot{\theta}$ and $\dot{\varphi}$ consist of a unique angular speed $\overline{\omega}$ passing by the fix point of the rigid. The support of this speed is in fact the axis of the instantaneous rotation movement for the rigid with fix point. The three movements of rotation does not succeed in turn as presented before, but in fact the rigid executes a succession of instantaneous rotation movement around the axis superposed on vector ω , the axis changing its position from one moment to another, passing permanently through the fix point of the rigid. This axis generated during the movement of the rigid, in rapport with the two systems of axes, two conical surfaces, one parabolic and one hyperbolic, permanently in contact after the axis of the rotation system movement.

It is compulsory to state that the line of the nodes O_1N which has been the axis Ox of the mobile system of axes around which circulated the mobile system with axes (and together with it also the rigid), remains permanently the intersection axis of the mobile plane Oxyz (connected to the rigid) with fix plan $Ox_1y_1z_1$. This axis is furthermore a mobile axis (instantaneous) of rotation (as in fact the axis of own Oz rotation), unlike fix axis Oz, which is the axis of precision movement.

So, it should be observed the fact that Euler's angles are rotation angles around a fix axis and around two mobile axes, from which one axis is fix of the mobile system and the other two are intersection lines of the perpendicular plan on this axis with the fix corresponding plan.

2. THE CASE OF HEAVY GYROSCOPE SITUATED IN PRECESSIONAL MOVEMENT

It should also be reminded the fact which is not sufficiently well indicated in the specialty literature, that the introduction of Euler's angles in the study of the rigid movement with a fix point is requested especially by the fix point. The most representative case under this aspect we believe to be the heavy gyroscope situated in precessional movement (fig.5).

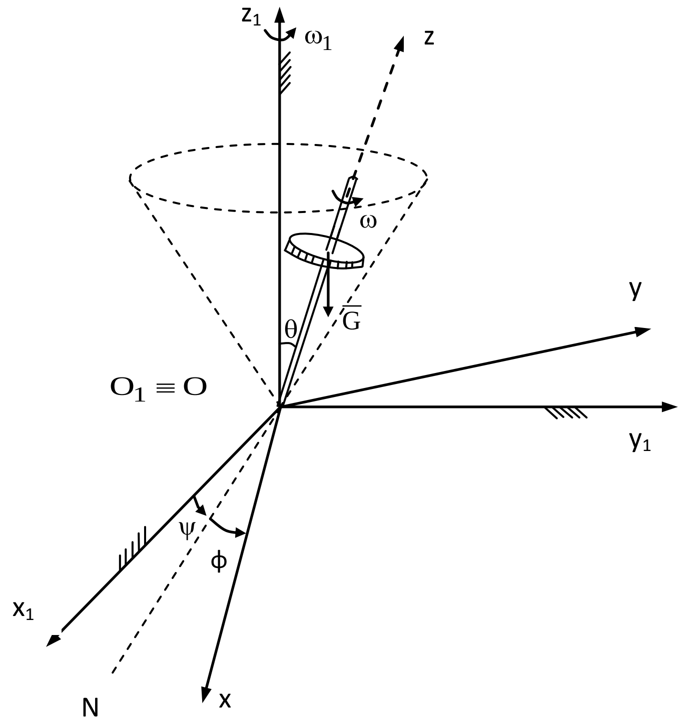

Figure 5. The heavy gyroscope situated in precessional movement

For this case of rotation movement of the rigid around own Oz symmetry axis with angular speed $\overline{\omega}$ as well as rotation around a vertical fix axis Oz, are accentuated like distinct movement other that in studying the movement by the introduction of Euler's angles appears like a necessity.

And, in the end, as it is known, the $\overline{\omega}$ vector of the instantaneous rotation of the rigid uses in its projections on the axes of the system of mobile or fix axes (rare). In the first case, it can be written

$$\overline{\omega} = \omega_x \bar{i} + \omega_y \bar{j} + \omega_z \overline{k} = \dot{\psi} \overline{k}_1 + \dot{\varphi} \overline{k} + \dot{\theta} \overline{\nu}, \quad (1)$$

$\overline{k}_1$, $\overline{k}$ and $\overline{\nu}$ unit vectors of the precessional movement axes (O_1Z_1), own rotation movement (OZ) and respectively nutation movement (O_1N).

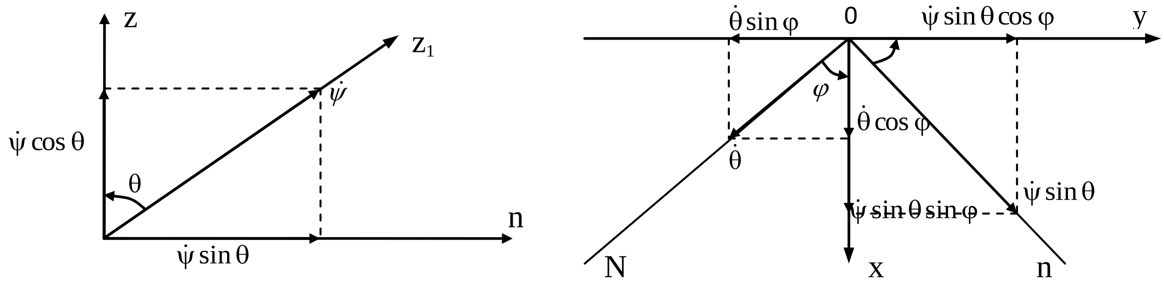

Figure 6

Observing the projections of the $\overline{\omega}$ vector from the relation (1) as can be seen in fig.6 results,

$$\begin{aligned} \omega_x &= \dot{\theta}\cos\varphi + \dot{\psi}\sin\theta\sin\varphi \\ \omega_y &= \dot{\psi}\sin\theta\cos\varphi - \dot{\theta}\sin\varphi \\ \omega_z &= \dot{\varphi} + \dot{\psi}\cos\theta \end{aligned} \tag{2}$$

Under the form (2) have been obtained the projections of the instantaneous revolving vector ω depending on Euler's angles in known form, that are so used in the study of the movement of the rigid with fix point.

3. CONCLUSIONS

In conclusion, the mobile system of axes, connected to the rigid in movement and initially situated superposed on the fix system of axes, in rapport with which can be studied the movement of the rigid, can be brought in a certain position, impressing successively three rotations around an axis passing by the fix point, with angles which are assumed to be precisely Euler's

References:

1. Appel, P. Traite de mecanique rationelle, Gauthier - Villars, Paris, 1984.
2 Mihut, N. M., Pasare, M. Mecanica, vol.III, Editura Sitech, Craiova, 2009. ISBN 978-606-530-158-0
3. Radoi, M., Deciu, E. M. Mecanica, Editura Didactica si Pedagogica, Bucureşti, 1977. ISBN 305IP13D1918ME
3. Silas, Gh., Grosanu, I., Mecanica, Editura Didactica si Pedagogica, Bucureşti, 1981. ISBN 146IPTIMME