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WAYS TO INCREASE THE CONTACT RATIO OF STRAIGHT BEVEL GEARS

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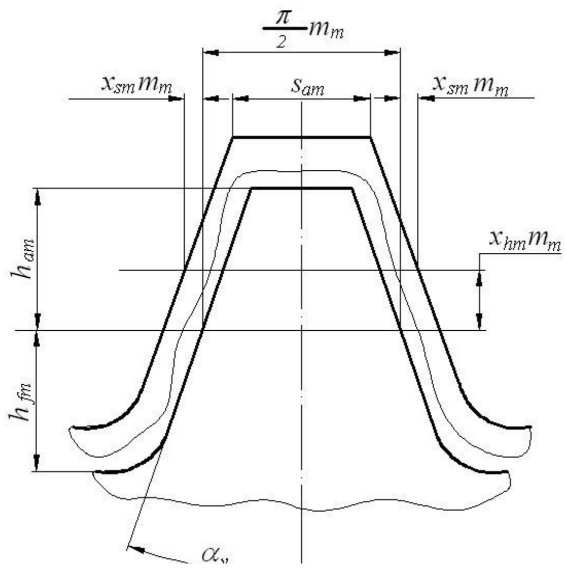
Abstract: The geometric and strength calculus of an octoidal straight bevel gear are made on a virtual cylindrical straight gear using Tredgold approximation. By analogy with cylindrical gears, it can be assumed that the main cause for the noise produced during the gearing process is determined also by the periodically change of the teeth stiffness, therefore by a contact ratio of $\varepsilon_{\alpha}<2$. This paper presents the ways to increase the contact ratio, so this to fit in the optimum range of $\varepsilon_{\upsilon\alpha} \in [1.95, 2.0]$, and terefore to obtain a quietness gearing process of bevel gears.

1. INTRODUCTION

The straight bevel gears are not strictly involute gears, the teeth flanks are made as flat surfaces instead of curved surfaces, to simplify the execution of the gear cutting tool. Therefore, the straight bevel gear is an *octoidal gear*, with the following features [2, 6]: the gearing line is an octoidal sphere; it can be executed only without profile shifts ($x_{hm1}=x_{hm2}=0$) or zero profile shifts ($x_{hm2}=-x_{hm1}$ and $x_{hm1}+x_{hm2}=0$) the pitch cones with reference cones coincide ($\delta_w=\delta$); the pressure angles on the pitch and reference cones coincide and in calculus it is used only pressure angle on the reference cones.



In the case of the straight bevel gears, in addition to the radial profile shift are also made thickness profile modification aimed to ensure an equal strength of wheels teeth to bending. Fig. 1 [1, 9] presents the rack displacement to obtain simultneaously radial profile shift x_{hm}, and thickness profile modification x_{sm}.

The contact ratio of bevel gear is determined on the virtual cylindrical gear, by *Tredgold* aproximation [2, 6, 9]. The geometrical and strength calculus relations are set by the ISO 10300 [9, 10, 11], ANSI/AGMA 2003-B97 [7] and DIN 3991 [8] standards.

In the papers [3] and [4] was determined and presented the periodically variation of teeth stiffness as being the main cause for the vibrations and noise occured during the geaoring process; this conclusion can be extended to straight bevel gears as the strength calculus and the teeth stiffness are determined on the virtual cylindrical gear. Similar to cylindrical gears, the influence of this cause can be reduced by using bevel gears charaterised by a high contact ratio in the range of $\varepsilon_{\upsilon\alpha} \in [1.95, 2.0]$. Thus, this paper presents the ways to obtain straight bevel gears with the above presented contact ratio.

2. THEORETICAL CONSIDERATIONS

The contact gear ratio of a straigth bevel gear is determined on the corresponding virtual gear using the following relation [5]

$$\varepsilon_{\upsilon\alpha} = \frac{\sqrt{d_{\upsilon a1}^2 - d_{\upsilon b1}^2} + \sqrt{d_{\upsilon a2}^2 - d_{\upsilon b2}^2} - 2a_{m\upsilon}\sin\alpha_\upsilon}{2\pi m_m \cos\alpha_\upsilon} \tag{1}$$

that depends on the tip and base diameters, and on the distance between the axes of the virtual gear coressponding to mean frontal cones.

As the straight bevel gear is made only without profile shifts or zero profile shifts, the tip diameters of the cylindrical virtual gear remain unchanged. These geometrical elements are determined with the following relations.

- The tip diameters of the virtual gears

$$d_{\upsilon a1} = d_{\upsilon 1} + 2h_{am1} = m_m\left[z_1\frac{\sqrt{u^2+1}}{u} + 2\left(h_a^* + x_{hm1}\right)\right]; \tag{2}$$

$$d_{\upsilon a2} = d_{\upsilon 2} + 2h_{am2} = m_m\left[z_1\sqrt{u^2+1} + 2\left(h_a^* - x_{hm1}\right)\right]; \tag{3}$$

- The base diameters of the virtual gears

$$d_{\upsilon b1} = d_{\upsilon 1}\cos\alpha = m_m z_1\frac{\sqrt{u^2+1}}{u}\cos\alpha_\upsilon \tag{4}$$

$$d_{\upsilon b2} = d_{\upsilon 2}\cos\alpha = m_m z_2\sqrt{u^2+1}\cos\alpha_\upsilon; \tag{5}$$

- The distance between the virtual gear axes

$$a_{m\upsilon} = \frac{m_m}{2}\sqrt{u^2+1}\left(\frac{z_1}{u} + z_2\right). \tag{6}$$

Using the relations (2)...(6), the relation (1) results in

$$\varepsilon_{\upsilon\alpha} = \varepsilon_{\upsilon\alpha z1} + \varepsilon_{\upsilon\alpha z2}, \tag{7}$$

Where the two partial contact ratios are determined as follows:

$$\varepsilon_{\upsilon\alpha z1} = \frac{1}{2\pi u\cos\alpha_\upsilon}\left[\sqrt{\left[z_1\sqrt{u^2+1} + 2u\left(h_{am}^* + x_{hm1}\right)\right]^2 - \left[z_1\sqrt{u^2+1}\cos\alpha_\upsilon\right]^2} - z_1\sqrt{u^2+1}\right]; \tag{8}$$

$$\varepsilon_{\upsilon\alpha z2} = \frac{1}{2\pi\cos\alpha_\upsilon}\left[\sqrt{\left[z_2\sqrt{u^2+1} + 2\left(h_{am}^* - x_{hm1}\right)\right]^2 - \left[z_2\sqrt{u^2+1}\cos\alpha_\upsilon\right]^2} - z_2\sqrt{u^2+1}\right], \tag{9}$$

These realtions are not dependent on the mean teeth module, thus resulting in a generalization of the relations and of the conclusions highlighted by this analysis.

The imposed restrictions are:

- avoiding the interference of the teeth

$$x_{hm1,2} \ge \left(z_{\min} - z_{1,2}\right)/z_{\min}, \text{ în care } z_{\min} = 2h_a^*/\sin^2\alpha; \tag{10}$$

- avoiding sharpening the teeth on the tip diameter

$$\frac{s_{am1}}{m_m} = \left[\left(inv\alpha_{\upsilon} - inv\alpha_{a\upsilon 1} \right) z_1 \frac{\sqrt{u^2+1}}{u} + \frac{s_{v1}}{m} \right] \frac{\cos\alpha_{\upsilon}}{\cos\alpha_{av1}} \geq 0.3 ; \qquad (11)$$

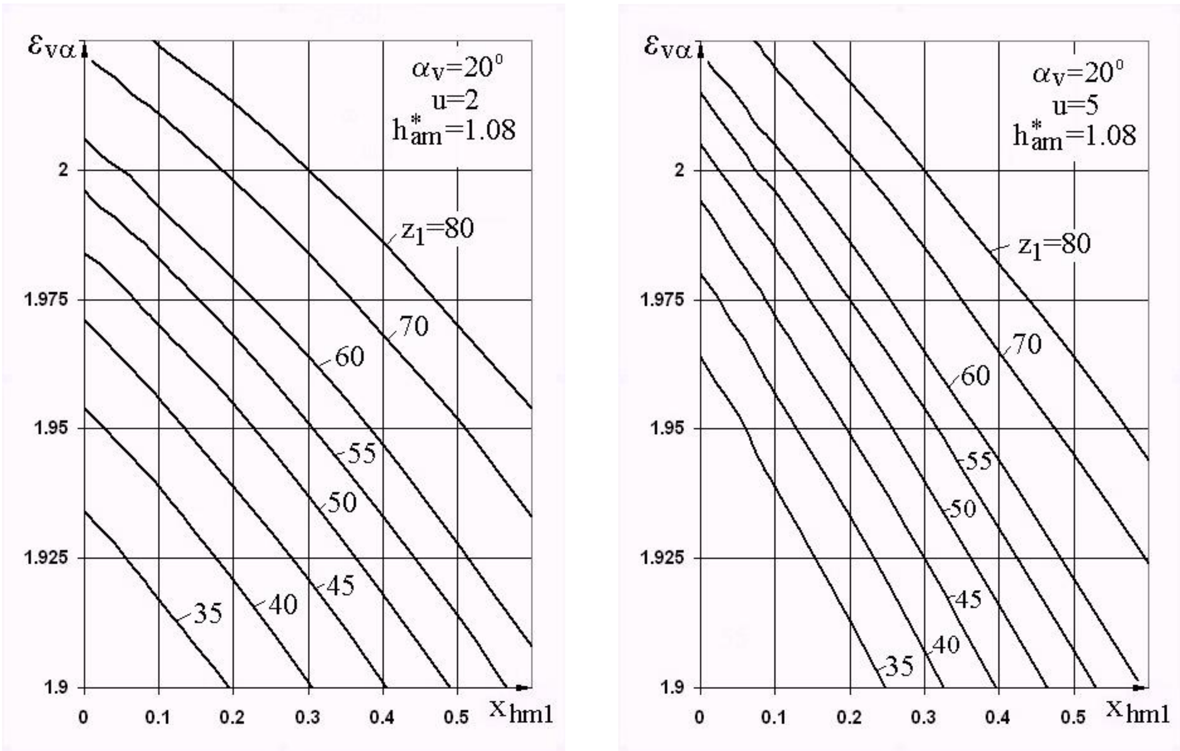
$$\frac{s_{am2}}{m_m} = \left[\left(inv\alpha_{\upsilon} - inv\alpha_{a\upsilon 2} \right) z_2 \sqrt{u^2+1} + \frac{s_{v2}}{m} \right] \frac{\cos\alpha_{\upsilon}}{\cos\alpha_{av2}} \geq 0.3 , \qquad (12)$$

where: $s_{\upsilon 1,2}/m_m = \pi/2 + 2x_{hm1,2} tg\alpha_{\upsilon}$ represents the width of the teeth on the mean pitch diameter; $\alpha_{a\upsilon 1} = \cos^{-1}\left[\frac{z_1\sqrt{u^2+1}\cos\alpha_{\upsilon}}{z_1\sqrt{u^2+1} + 2u\left(h_{am}^{*} + x_{hm1}\right)} \right]$ and $\alpha_{a\upsilon 2} = \cos^{-1}\left[\frac{z_2\sqrt{u^2+1}\cos\alpha_{\upsilon}}{z_2\sqrt{u^2+1} + 2\left(h_{am}^{*} - x_{hm1}\right)} \right]$ are the pressure angles on the tip of the virtual teeth.

3. CALCULUS AND CONCLUSIONS

The analysis of the possibilities to increase the contact gear ratio of the straight bevel gears is made considering the diagrams obtained by running a calulus program developed by the authors, using the above presented realtions.

Figure 2 presents the variation of the gear contact ratio and the optimum range in which the straight bevel gear generates low noise. The characteristics of this specific straight bevel gear are presented in the following diagrams.



a *b*

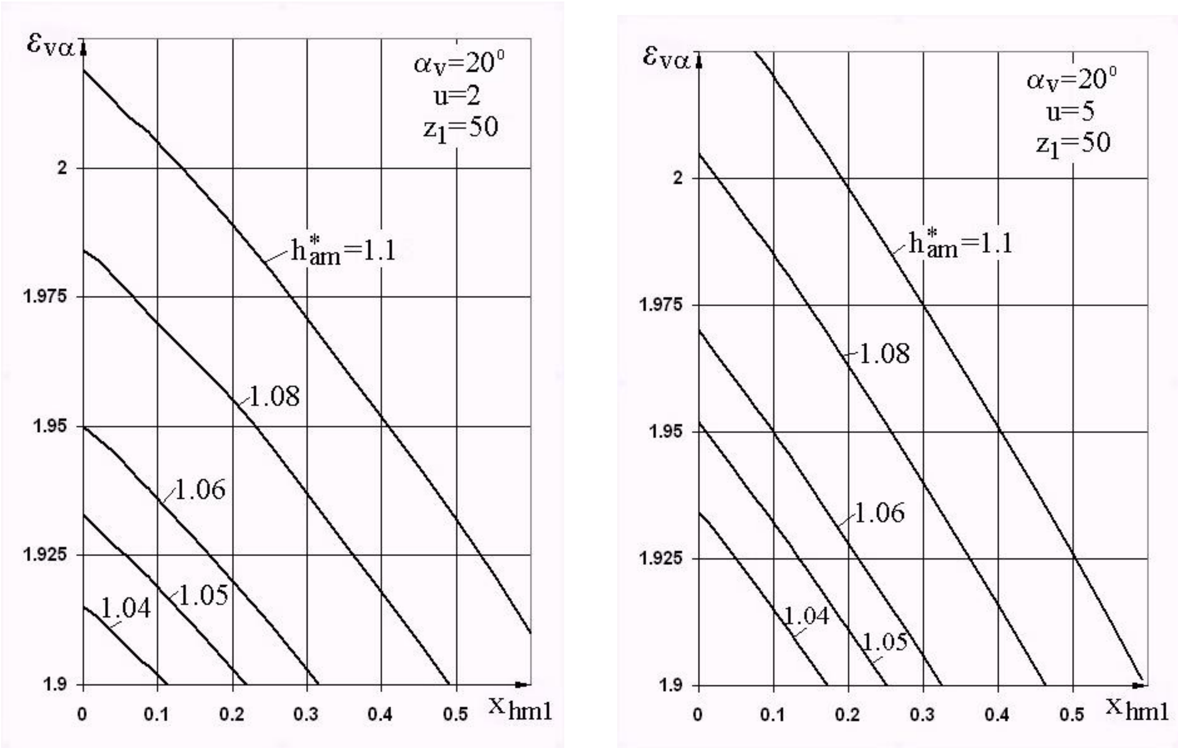
Fig. 2. ***$\varepsilon_{\upsilon\alpha}$ = f(x_{hm1}, z_1, u, α_{υ} = 20º)***

The analysis of the diagrams presented in Fig. 2 highlights the following conclusions:

- The transverse contact ratio of the straight bevel gear $\varepsilon_{\upsilon\alpha}$ decreases as the radial profile shift coefficient x_{hm1} increases and is increasing as the number of the teeth of pinion and the gear ratio *u* increase;

- In the case of a standard value of the teeth tip coefficient $x_{hm1} \leq 1.0$, the contact ratio is $\varepsilon_{\upsilon\alpha} < 1.95$ for rack profile angle of $\alpha_\upsilon = 20^o$; thus, no straigth bevel gear made with a rack characterised by standard parameteres ($\alpha_\upsilon = 20^o$, $h^*_{am} = 1.0$ şi $c^* \leq 0.25$) is not fitting in the range of $\varepsilon_{\upsilon\alpha} \in [1.95, 2.0]$;
- The value of the contact ratio of straight bevel gear is in the range of $\varepsilon_{\upsilon\alpha} \in [1.95, 2.0]$ for a teeth number $z_1 \geq 40$, if $u=2$, respectivly for a teeth number of $z_1 \geq 35$, if $u=5$, for a mean addendum coefficient $h^*_{am} = 1.08$; under this value of h^*_{am}, the contact ratio is $\varepsilon_{\upsilon\alpha} < 1.95$.

Figure 3 presents the variation of the contact ratio depending on the value of the radial profile shift coefficient x_{hm1} and of the mean addendum coefficient h^*_{am}. The following conclusions can be drawn:



a *b*

Fig. 3. $\varepsilon_{\upsilon\alpha} = f(x_{hm1}, h^*_{am}, u, z_1 = 50, \alpha = 20^o)$

- The transerve contact ratio of straight bevel gear increases with the decrease of the radial profil shift displacement x_{hm1}, and with the increase of gear ratio u and the mean addendum coefficient h^*_{am};
- Considering the two diagrams, it can be noticed that for small gear ratios (u=2, fig. 3, a) the transverse contact ratio is in the optimum range $\varepsilon_{\upsilon\alpha} \in [1.95, 2.0]$ for mean addendum coefficient $h^*_{am} > 1.06$. Instead, for high gear ratios (u=5, fig. 3, b), the transverse contact ratio is in the optimum range for mean addendum coefficient of $h^*_{am} = 1.05$;

- For a higher number of teeth of pinion (z_1=50, fig. 3), it can be obtained a contact ratio fitted in the optimum range even for small values of radial profile shift coefficient x_{hm1}.

Figure 4 presents the variation of contact ratio as a function of the radial profile shift coefficient x_{hm1} and the number of teeth of pinion z_1, for the a specific value of pressure angle 17.5º.

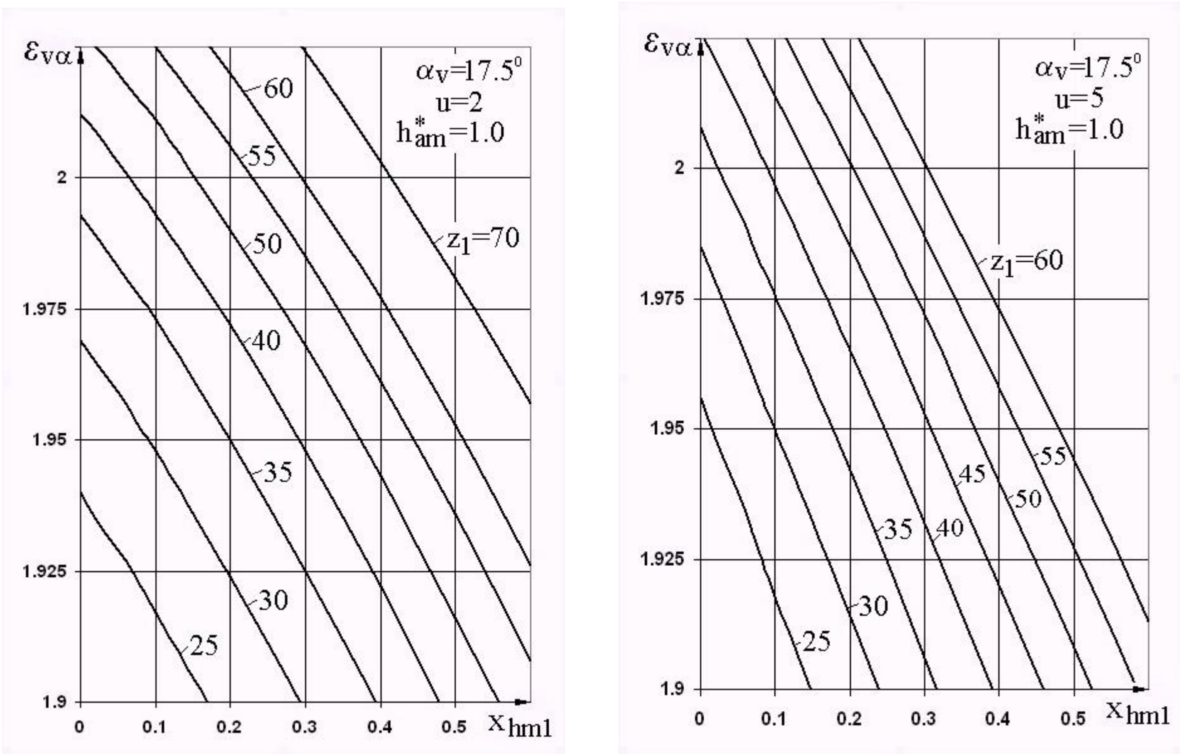


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Fig. 4. $\varepsilon_{\upsilon\alpha} = f(x_{hm1}, z_1, u, \alpha_\upsilon = 17.5^o)$

The following conclusions can be drawn:

- The decrease of the pressure angle has a significant influence on the decrease of the number of teeth of pinion for wgich the contact ratio is fiited in the range of $\varepsilon_{\upsilon\alpha} \in [1.95, 2.0]$;
- For small gear ratios (u=2, fig. 4, a), the number of teeth of pinion for ehich the contact ratio is in the optimum range is z_1=35, compared to the case of pressure angle 20º where z_1=40 and $x_{hm1}\approx 0$ (v. fig. 2, a);
- For small gear ratio (u=5, fig. 4, b), the number of teeth of pinion decreases more (z_1=20, fig. 4, b), compared to the case of pressure angle of 20º (in this situation z_1=35, fig. 2, b).

Concluding, by choosing the adequate number of teeth of straight bevel pinion, of pressure angle α_υ, of mean addendum coefficient h^*_{am} and of the profile shift coefficient x_{hm1} can be obtained straight bevel gears that fit to the optimum range of contact ratio $\varepsilon_{\upsilon\alpha} \in [1.95, 2.0]$, resulting so in low noise operation.

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