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MATHEMATICAL MODEL FOR CALCULATION OF LINEAR ELASTIC SHAFTS

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Abstract: Spatial mechanisms of different families of zero is undefined static systems, order of determination is given by the number representing the family mechanism. Transmission shaft, the simplest cinematic form, is a third family RRRR mechanism. The method calls for the relative displacements and establishes the mathematical model that allows linear elastic calculation first and then simple transmission cross shaft with symmetric transmission.

1. INTRODUCTION

Mechanisms, which belong to different families of zero, are systems [8] static indefinite. Thus, for example, the mechanism $4R$ third family plan, $ABCD$ with joints A, D, fixed mechanism situated in the OXY, can write $6x3=18$ balance equations and the number of unknowns is $5x4=20$.

If this example of the equations can determine reaction forces R_{AX}, R_{AY} and momentum M_{AZ} from joint A but can not determine the reaction force R_{AZ} and moments M_{AX}, M_{AY}. To determine these components make use of linear elastic calculation. An analogous situation occurs in a spatial mechanism $4R$, $ABCD$ with concurrent axes O, mechanism represented by full lines in Figure 1. It is also a third mechanism family and is a statically indeterminate system and the degree of indeterminacy is amplified when using the cross shaft $BB'CC'$ (Fig. 1).

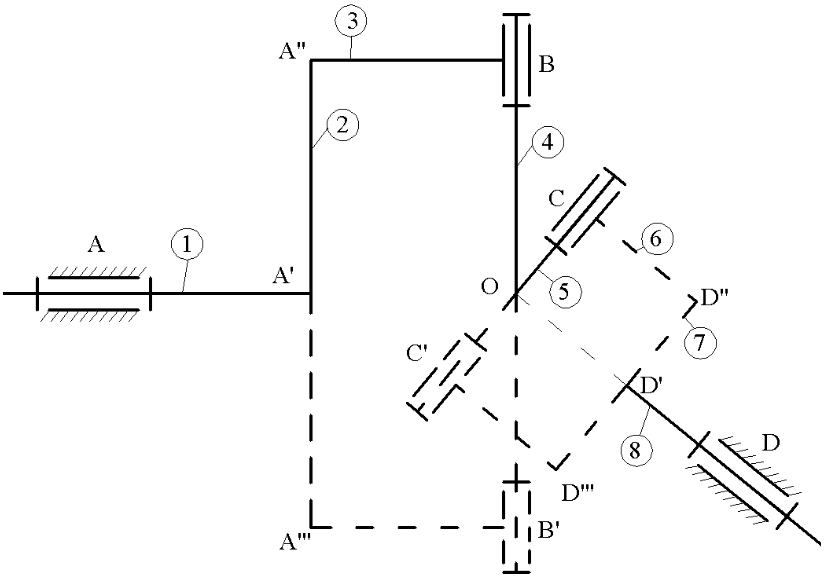

Figure 1. Transmission cross shaft - constructive elements

The following will develop mathematical model to calculate the reaction forces for both linear elastic mechanism represented by full lines in Fig. 1 as the one represented by the dotted line, that provided the cross shaft.

The calculation method is based on relative displacements, method presented in [8] plùckeriene coordinate expression.

2. RELATIONS BETWEEN RELATIVE DISPLACEMENTS AND EFFORTS AT THE ENDS OF A BAR

It is considered right bar AB (fig. 2) of length l, constant-area section 7, modules of elasticity A and E,G be a local reference system, the central axis Ay, Az being the principal axes of inertia of normal section through A

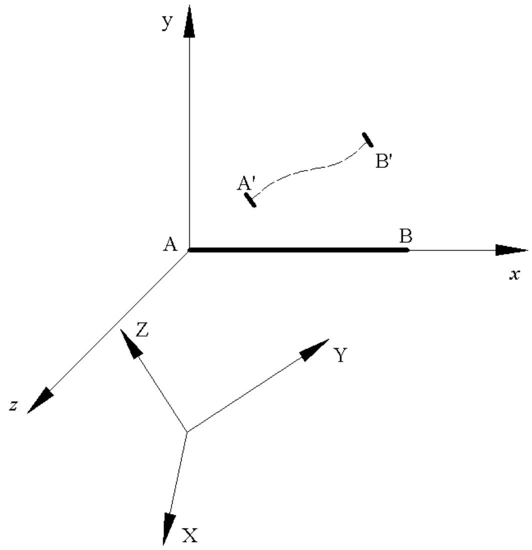

Figure 2. Element** AB**, represented in the local and the general reference

Under the influence efforts to end $\{f_A\},\{f_B\}$ a bar sections A and B have d_A, d_B movements, efforts and expressed in coordinated movements plùckeriene.

$$\{f_A\}=[f_{Ax}\quad f_{Ay}\quad f_{Az}\quad m_{Ax}\quad m_{Ay}\quad m_{Az}]^T;\ \{f_B\}=[f_{Bx}\quad f_{By}\quad f_{Bz}\quad m_{Bx}\quad m_{By}\quad m_{Bz}]^T \qquad (2.1)$$

$$\{d_A\}=[\theta_{Ax}\quad \theta_{Ay}\quad \theta_{Az}\quad d_{Ax}\quad d_{Ay}\quad d_{Az}]^T;\{d_B\}=[\theta_{Bx}\quad \theta_{By}\quad \theta_{Bz}\quad d_{Bx}\quad d_{By}\quad d_{Bz}]^T \qquad (2.2)$$

where $\bar{\theta}_A(\theta_{Ax}\quad \theta_{Ay}\quad \theta_{Az}), \bar{\theta}_B(\theta_{Bx}\quad \theta_{By}\quad \theta_{Bz})$ is small rotations of sections, $\bar{d}_A(d_{Ax}\quad d_{Ay}\quad d_{Az}), \bar{d}_B(d_{Bx}\quad d_{By}\quad d_{Bz})$ are displacements of points A and B, $\bar{f}_A(f_{Ax}\quad f_{Ay}\quad f_{Az}), \bar{f}_B(f_{Bx}\quad f_{By}\quad f_{Bz})$ are forces effort, $\bar{m}_A(m_{Ax}\quad m_{Ay}\quad m_{Az})$, , $\bar{m}_B(m_{Bx}\quad m_{By}\quad m_{Bz})$ are moments effortlessly checking relations efforts and movements

$$\{f_A\}+\{f_B\}=\{0\} \qquad (2.3)$$

$$\bar{d}_A = \overline{AA'}\ ;\ \bar{d}_B = \overline{BB'} + \overline{AB}\,x\,\bar{\theta}_B \qquad (2.4)$$

Stiffness matrix of flexibility matrix $[k_{AB}]$ and $[h_{AB}]=[k_{AB}]^{-1}$, I_y, I_z being the main central moments of inertia and moment of inertia I_x equivalent request for twisting their expressions [8]

$$[k_{AB}]=\begin{bmatrix} 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ 0 & 0 & \frac{6EI_z}{l^2} & 0 & \frac{12EI_z}{l^3} & 0 \\ 0 & -\frac{6EI_y}{l^2} & 0 & 0 & 0 & \frac{12EI_y}{l^3} \\ \frac{GI_x}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{4EI_y}{l} & 0 & 0 & 0 & -\frac{6EIy}{l^2} \\ 0 & 0 & \frac{4EI_z}{l} & 0 & \frac{6EI_z}{l^2} & 0 \end{bmatrix};$$

$$[h_{AB}]=\begin{bmatrix} 0 & 0 & 0 & \frac{l}{GI_x} & 0 & 0 \\ 0 & 0 & \frac{l^2}{2EI_y} & 0 & \frac{l}{EI_y} & 0 \\ 0 & -\frac{l^2}{2EI_z} & 0 & 0 & 0 & \frac{l}{EI_z} \\ \frac{l}{EA} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{l^3}{3EI_z} & 0 & 0 & 0 & -\frac{l^2}{2EI_z} \\ 0 & 0 & \frac{l^3}{3EI_y} & 0 & \frac{l^2}{2EI_y} & 0 \end{bmatrix} \tag{2.5}$$

are obtained [8] relations

$$\{f_A\}=[k_{AB}]\{d_{AB}\};\ \{d_{AB}\}=[h_{AB}]\{f_A\} \tag{2.6}$$

where $\{d_{AB}\}$ is the relative displacement

$$\{d_{AB}\}=\{d_A\}-\{d_B\} \tag{2.7}$$

To move to a general reference system (fig. 2) using the notation

- (X_A, Y_A, Z_A)-coordinates of point A in system $OXYZ$
- $(\alpha_i, \beta_i, \gamma_i)$, $i=1,2,3$ - cosines directors of axes Ax, Ay, Az
- $[G_{AB}],[R_{AB}],[T_{AB}],[T_{AB}]^{-1}$, matrices defined by relations

$$[G_{AB}]=\begin{bmatrix} 0 & -Z_A & Y_A \\ Z_A & 0 & -X_A \\ -Y_A & X_A & 0 \end{bmatrix};\ [R_{AB}]=\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \tag{2.8}$$

$$[G_{AB}]=\begin{bmatrix} 0 & -Z_A & Y_A \\ Z_A & 0 & -X_A \\ -Y_A & X_A & 0 \end{bmatrix};\ [R_{AB}]=\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \tag{2.9}$$

and in order to obtain the equalities

$$\{\Delta_A\}=[T_{AB}]\{d_A\};\ \{\Delta_B\}=[T_{AB}]\{d_B\};\ \{\Delta_{AB}\}=\{\Delta_A\}-\{\Delta_B\} \tag{2.10}$$

$$\{F_A\}=[T_{AB}]\{f_A\};\ \{F_B\}=[T_{AB}]\{f_B\} \tag{2.11}$$

$$\{\Delta_A\} = [T_{AB}]\{d_A\}\ ;\ \{\Delta_B\} = [T_{AB}]\{d_B\};\ \{\Delta_{AB}\} = \{\Delta_A\} - \{\Delta_B\} \quad (2.12)$$

$$\{F_A\} = [T_{AB}]\{f_A\}\ ;\ \{F_B\} = [T_{AB}]\{f_B\} \quad (2.13)$$

For an angled bar $ABCD$ is obtained flexibility matrix [8]

$$\{\Delta_A\} = [T_{AB}]\{d_A\}\ ;\ \{\Delta_B\} = [T_{AB}]\{d_B\};\ \{\Delta_{AB}\} = \{\Delta_A\} - \{\Delta_B\} \quad (2.14)$$

CONCLUSIONS

Using relative displacements and coordinates plùckeriene [8] makes it possible to develop mathematical model to determine the mechanisms gimballed reaction forces without technological deviations.

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