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PRE-TEETHING WITH PINION SHAPED CUTTER OF GEAR WHEELS WITH NON-EVOLVENTIC

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Abstract Processing teething by copying method presents special disadvantages, as it introduces dividing errors unacceptable for the gear cinematic conditions That is why, where possible, it is preferred processing by rolling: finishing by shaver cutting; pre-finishing with worm cutter or pinion shaped cutter.

To determine the joint surfaces the following algorithm for the problem will be implemented:

- motion law of engagement point is chosen in the immobile space (connected to the axes of the gear wheels);
- characteristics lines are found in rolling coordinates systems, connected to the two wheels (contact lines) and immobile system (gearing line);
- it is envisaged that, when contact occurs, the surfaces should not intersect in the limits of the portions used as active flanks of the teeth;

- through teeth contact lines are drawn surfaces which, must satisfy the conditions in order to be conjugated surfaces (common normal at contact surfaces, in all points of contact lines; is perpendicular to the relative speed vector of conjugated points and on the tangent to contact lines; form of conjugated surface sand position of contact lines must ensure the requirement that surface curvature in normal plan to the contact line should be at least equal to the geodesic curvature of contact line.

$$\frac{1}{\tau_g} \geq \frac{1}{2} \cdot \left(\frac{1}{\rho_1} \cdot \sin 2\varphi - \frac{1}{\rho_2} \cdot \sin 2\varphi \right)$$

$$\frac{1}{r} \cdot \cos\theta = \left(\frac{1}{\rho_1} \cdot \cos^2 \varphi - \frac{1}{\rho_2} \cdot \sin^2 \varphi \right)$$

According to figure 1, notations have the following meanings:

$\frac{1}{\rho_1}; \frac{1}{\rho_2}$ – main curvatures of conjugated surfaces;

$\frac{1}{\tau_g}$ – geodesic curvature of contact line;

$\frac{1}{r}$ – curvature of contact line;

θ – angle between normal to surface and main normal to the contact line;

φ – angle between direction and tanget to curve having main curvature.

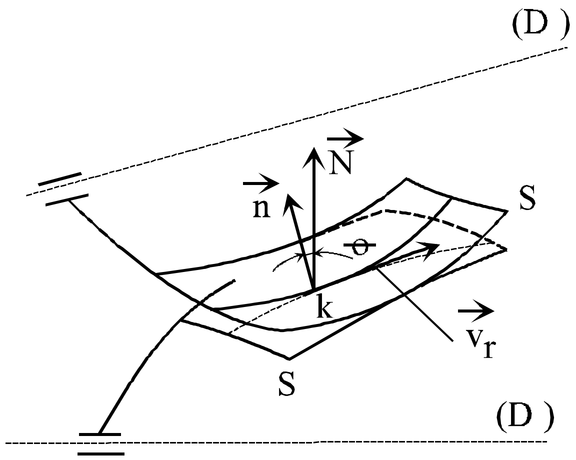

Figure 1. Elements of conjugated surfaces in crossed axes gearing

A modern method for determining shaver tooth profile that must process the teeth of a wheel whose profile is given and which leads to outstanding technological facilities, is based on the mathematical theory of envelope surfaces.

In the case of gearing with crossed axes, complementary generating gear racks, with inclined teeth have relative movement such that their reference planes and flanks slip to each other. Reference planes perform translational movements, rolling without slipping on wheel rolling cylinders of the gear, thus obtaining two separate gear cylindrical wheel – gear rack with inclined teeth.

Figure 2 shows how the shift from gear of two wheels with crossed axes to the equivalent gears, cylindrical wheel - rack with inclined teeth.

Each rack has a translational motion with velocity v = □ · r (where □, r is the angular velocity, respectively the rolling cylinder radius for the gear wheel considered), as a result of rolling without sliding over the wheel rolling cylinders. In order for engagement to take place, the rack teeth sidewalls profiles (both the rack teeth of processing wheel and also the pinion shaped cutter, in normal section on tooth direction should overlap, and the relative speed of the flanks in this section should be void.

Because between the speed direction $\vec{v_1}$ şi $\vec{v_2}$ there is an angle □ equal to the crossing angle of wheel axis there appears, along the teeth a relative, sliding motion, with the speed:

$$\vec{v_{r21}} = \vec{v_2} - \vec{v_1}$$

corresponding to the contact overlapped points of the two flanks.

Since the reference rack gears with inclined teeth (having the inclined angle of the teeth equal to the angle of inclination of the teeth wheels that is associated to the rack) have overlapping profiles in the normal section on the tooth direction, determining the active profile of the conjugated wheel (pinion shaped cutter) - if one wheel profile is known of the gear wheels (processing wheel) - is realized by the following algorithm:

The corresponding gear rack profile afferent to the gear wheel is determined in section normal to its axis, is determined the gear rack profile corresponding to the shaver in normal section on its axis, profile resulting from the gear rack profile corresponding to the processing wheel (both racks have overlapping profiles in the normal section on the tooth direction); shaver profile is determined from its corresponding rack profile.

Note that in this case, the conjugated gear wheel profile (pinion shaped cutter) is determined in normal section to its axis.

Based on the mathematical theory of enveloping surfaces, if it is given a family of curves C□ dependent on □ parameter, defined by the equation:

$$F(x, y, \varphi) = 0$$

it admits an envelope curve if there is a curve □ which does not belong to the family and checks the following conditions:

- to each curve (□ corresponds to a point M on curve □, and reciprocal to each point on curve □ corresponds a family curve;
- curves C□ and □ are tangent in point M;
- there are no common arches between □ and curves C□.

Hence, the envelope equation can be derived from solving the system:

$$\begin{cases} F(x, y, \varphi) = 0 \\ F' \varphi(x, y, \varphi) = 0 \end{cases}$$

If removed from this system parameter □ curve □equation is obtained as the default, and if you can not remove parameter □ envelope equation is obtained in a parametric form. It should be noted that following the full system resolution we can get the coordinates of singular point of curves C□.

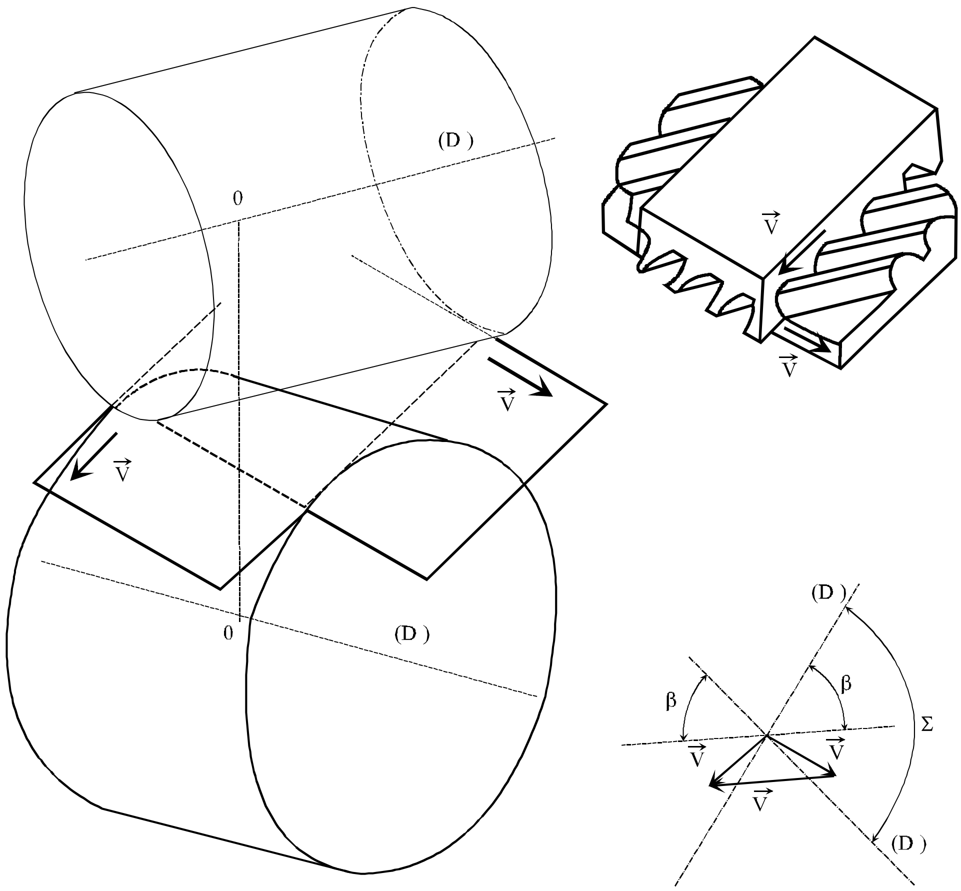

Figure 2. Gearing with crossed axes and gears equivalent to the gear rack cylindrical wheel

With all these general considerations we can proceed to determine the profiles for rack and pinion shaped cutter. If we consider two reference systems xAy attached to wheel and xPy attached to the rack connected between them by the independent parameter □ (which is the rotation angle of the wheel), knowing the C profile of the tooth wheel in normal section to its axis, as a result of switching to xPy reference system, considering the rolling without slipping of Px axis over the circle radius r_1, we obtain the family of curves equation $C_□$ in the reference system xPy, under the form of an equation like this:

$$F(x, y, \varphi) = 0$$

By solving the system:

$$\begin{cases} F(x, y, \varphi) = 0 \\ F'\varphi(x, y, \varphi) = 0 \end{cases}$$

curve □ equation is obtained, the reference system xPy.

On the basis of the same reasoning, if we know the rack profile equation in system xPy, we can deduce the wheel profile equation in system xAy.

According to figure 3 the connection between the coordinates of a certain point N in the two reference systems can be inferred as follows:

$$\begin{cases} x_N = x_A + X_N \cdot \cos\varphi + Y_N \cdot \sin\varphi \\ y_N = y_A + Y_N \cdot \cos\varphi - X_N \cdot \sin\varphi \end{cases} (*)$$

reciprocal can be written as:

$$\begin{cases} X_N = (x_N - x_A) \cdot \cos\varphi - (y_N - y_A) \cdot \sin\varphi \\ Y_N = (x_N - x_A) \cdot \sin\varphi + (y_N - y_A) \cdot \cos\varphi \end{cases} (**)$$

They represent the link between the coordinates of a certain point in two reference systems rotated and moved. xA and y_A are the coordinates of point A (center of reference system xAy) in reference system xPy. As the circle of radius "r" rolls without slipping over axis P_X, segment PP 'has length □ · □. It result coordinates xA and y_A as given by relations:

$$\begin{cases} x_A = \Gamma \cdot (\varphi - \sin\varphi) \\ y_A = \Gamma \cdot (1 - \cos\varphi) \end{cases}$$

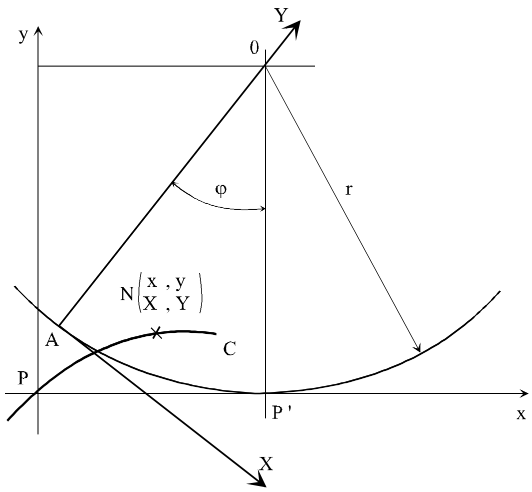

Figure 3. Reference systems related to the wheel and rack gear

So, the coordinates of a certain point on the C profile of the tooth or on the □ profile of the rack in the rack-related reference system, respectively the wheel, will be:

$$\begin{cases} x_c = r\cdot(\varphi - \sin\varphi) + X_c\cdot\cos\varphi + Y_c\cdot\sin\varphi \\ y_c = r\cdot(1-\cos\varphi) + Y_c\cdot\cos\varphi - X_c\cdot\sin\varphi \end{cases}$$

iar:

$$X_\gamma = \left[x_\gamma - r\cdot(\varphi - \sin\varphi)\right]\cdot\cos\varphi - \left[y_\gamma - r\cdot(1-\cos\varphi)\right]\cdot\sin\varphi$$

$$Y_\gamma = \left[x_\gamma - r\cdot(\varphi - \sin\varphi)\right]\cdot\sin\varphi - \left[y_\gamma - r\cdot(1-\cos\varphi)\right]\cdot\cos\varphi$$

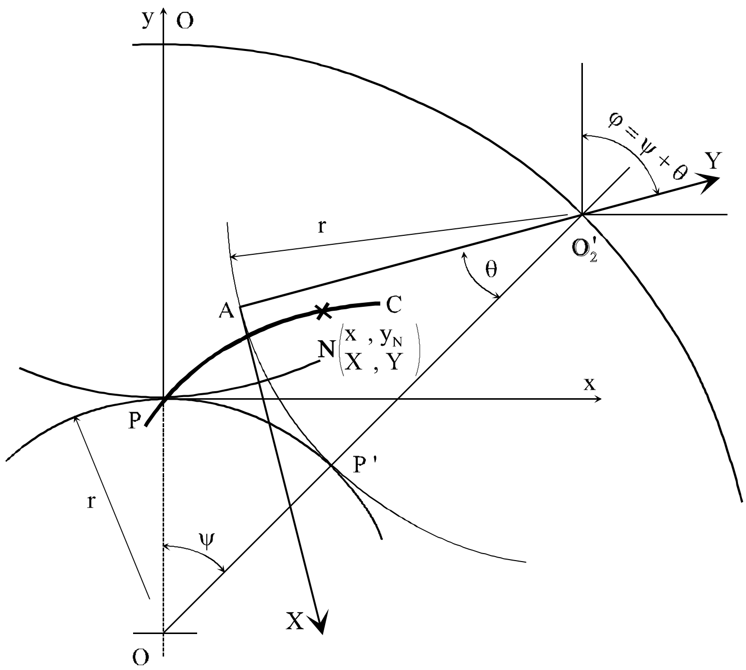

Figure 4. Reference systems related to the wheel and gear rack

Based on this algorithm is determined easily the combined profile, for pinion shaped cutter, for wheel profile processing. For checking to obtain a correct combined profile we can make an analysis on the equivalent gear. The study of equivalent gear allows in its turn direct determination of the pinion shaped cutter profile.

In the case of equivalent gear analysis, after establishing the equivalent rolling diameters we can pass to establish the combined profiles by the same method as described above. To the wheel whose profile is known, will be linked reference system xAy, and to the pinion shaped cutter whose profiler should be determined, will be linked reference system xPy. Note that profiles entering the calculation are profiles on the teeth in their normal section. According to Figure 4 we can determine the connection between the coordinates of a point N in the two reference systems.

For establishing the link between the coordinates of point N in the two reference systems we use system equations (*), (**). Coordinates of point A(x_A, y_A) are determined by taking into account the rolling circle radius r_1 şi r_s. Between rotation angles □ and □ there is the link:

$$r_1\cdot\psi = r_s\cdot\theta; \quad \text{or} \quad \theta = \frac{r_1}{r_s}\cdot\psi$$

From the figure result the coordinates x_A and y_A of point A:

$$x_A = (r_1 + r_s) \cdot \sin\psi - r_s \cdot \sin(\theta + \psi)$$
$$y_A = (r_1 + r_s) \cdot \cos\psi - r_1 - r_s \cdot \cos(\theta + \psi)$$

As a result, the link between the coordinates of point N in the two reference systems according to wheel rolling angle 1 will be:

$$x_N = (r_1 + r_s) \cdot \sin\psi + (Y_N - r_s) \cdot \sin\left(\frac{r_1 + r_s}{r_s} \cdot \psi\right) + X_N \cdot \cos\left(\frac{r_1 + r_s}{r_s} \cdot \psi\right)$$
$$y_N = (r_1 + r_s) \cdot \cos\psi + (Y_N - r_s) \cdot \cos\left(\frac{r_1 + r_s}{r_s} \cdot \psi\right) - X_N \cdot \sin\left(\frac{r_1 + r_s}{r_s} \cdot \psi\right)$$

Following these considerations it can be inferred a uniform system for determining profiles has been set up to determine the combined cylindrical gear wheels profiles which engage the crossed axes. It can also be easily established the profile of rack gears wheels, which allow profiling the worm-cutter, the pinion shaped cutter, or the shaver for processing by rolling a given profile. Also it can also be determined the pinion shaped cutter profile for groove processing.

The profiles of grooved wheel – pinion shaped cutter profiles gear result from this as they are presented in figure 5.

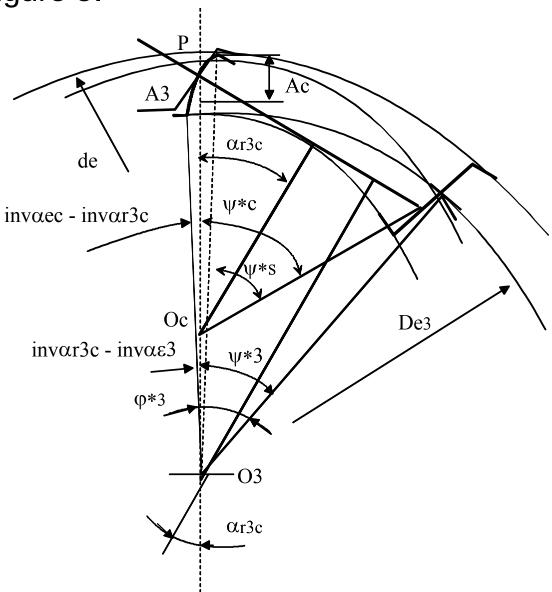

Figure 5. Profile elements conjugated in grooved wheel – pinion shaped cutter gearing

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