MOTION LAWS OF DISPERSIVE PRISMS

Bodea Renata [1], Gruescu Corina [2]
[1] University of Oradea, [2]University "Politehnica" of Timisoara,
prod.conf@rdslink.ro, corina.gruescu@mec.upt.ro

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Abstract: The paper refers to the motion law of a dispersive prism (regarded as the dispersive element in the scheme of a monochromator). A sort of glass highly dispersive within the visible range was chosen from the Schott Catalogue 2012. The refractive indexes were computed using Sellmayer's formula for the range of wavelengths (380...780) nm. The values of the incidence angle for the constant minimum deviation angle of 61.11^o were computed in relationship with the incidence angle. The nonlinear function $\varepsilon_1(\lambda)$ – expressing the motion law of the prism – was interpolated by a polynomial 6 and three linear functions respectively.

1. INTRODUCTION

Spectral apparatuses use monochromatic radiation, which is provided by the monochromator. The dispersive element is the optical part which actually splits white light into monochromatic radiations. The dispersive element can be a special prism or a reflecting/refractive grating. The most common dispersive prism (fig. 1) should be manufactured with a highly dispersive sort of glass and feature a large angle α.

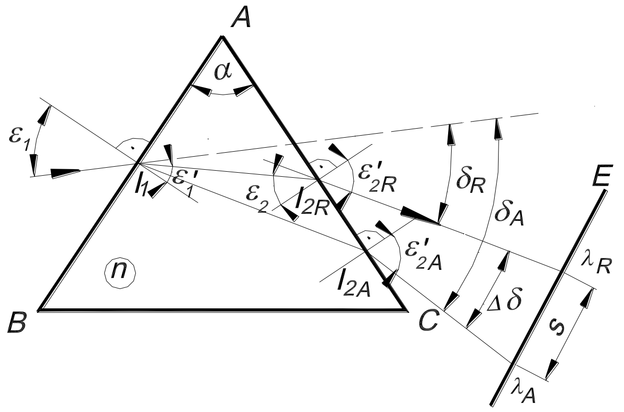

Figure 1. Dispersive prism (α - active angle of the prism, n – refractive index of glass, ε_1 – incidence angle, δ_R – deviation of red radiation, δ_A – deviation of blue radiation, $\Delta\delta$ - angular dispersion of the prism, s – linear dispersion of the prism)

For each wavelength are valid the following relationships [1-3]:

$$\sin\varepsilon_1 = n\sin\varepsilon_1^{'}, \quad (1.1)$$

$$n\sin\varepsilon_2 = \sin\varepsilon_2^{'}, \quad (1.2)$$

$$\alpha = \varepsilon_1^{'} + \varepsilon_2, \quad (1.3)$$

$$\delta = \varepsilon_1 + \varepsilon_2^{'} - \alpha. \quad (1.4)$$

where ε_1 – incidence angle on the first surface, n – referactive index depending on the wavelength, ε'_1 – refraction angle from the first surface, ε_2 – incidence angle on the second suface, ε'_2 – refraction angle from the second surface, α - active angle of the prism, δ - deviation angle of the prism.

Considering the visible range of wavelengths, the dispersion angle of the prism $\Delta\delta$ is computed as difference between the deviations of the marginal wavelengths (i.e. blue and red radiations):

$$\Delta\delta = \delta_A - \delta_R, \tag{1.5}$$

where δ_A – deviation of the marginal blue ray, δ_R – deviation of the marginal red ray.

2. PRINCIPLE OF ROTATING PRISM IN SPECTRAL APPARATUSES

The basic scheme of a spectral apparatus, working with transmitted light, is presented in figure 2 [4].

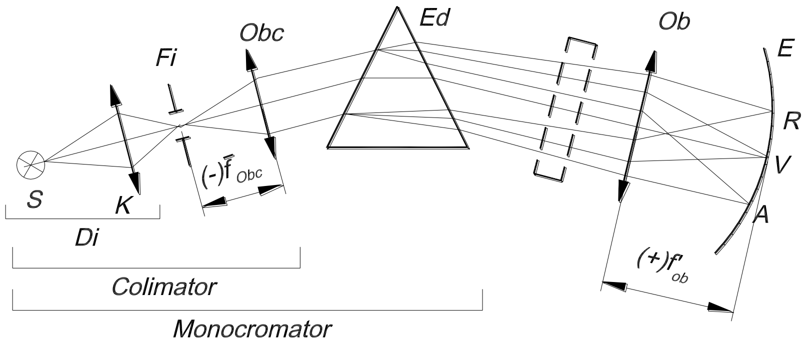

Figure 2 Basic scheme of a spectral apparatus

The white light comes from the illumination device Di (consisting of the source S and condenser K), passes through the rectangle-shaped entrance slit F_i toward the collimation objective Obc. The parallel beam falls over the prism Ed, which disperses light. Monochromatic radiations pass through a liquid sample and are focused by the objective Ob on the exit slit V. In relationship with the incidence angle, i.e. the prism position, on the exit slit is projected a certain wavelength. The other wavelengths are absorbed by the black interior wall of the apparatus E.

In order to get each wavelength, one by one, on the exit slit, a slight rotation of the prism is necessary.

In practice, the rotation is obtained by means of cam mechanisms or screws and high ratio transmissions. The most difficult task is establishing the motion law of the prism, i.e. the function $\varepsilon_1(\lambda)$.

3. MOTION LAW OF A DISPERSIVE PRISM

Dispersive prisms work around the minimum deviation angle. In this case, the equations (1...4) take a specific form:

$$\varepsilon_1 = \varepsilon_2', \tag{3.1}$$

$$\varepsilon_1' = \varepsilon_2 = \frac{\alpha}{2}, \tag{3.2}$$

$$\delta = 2\varepsilon_1 - \alpha = 2\varepsilon_2' - \alpha, \tag{3.3}$$

$$\sin\varepsilon_1 = n\sin\varepsilon_1' = n\sin\varepsilon_2 = \sin\varepsilon_2'. \tag{3.4}$$

Processing the above equations, the expression of the deviation angle in respect with the incidence angle is:

$$\delta = \varepsilon_1 + \arcsin\left\{ n \sin\left[\alpha - \arcsin\left(\frac{\sin\varepsilon_1}{n} \right) \right] - \alpha \right\}. \tag{3.5}$$

Considering known the minimum deviation δ, the active angle of the prism α and the refractive index n of the glass, the equation (3.5) contains only one unknown variable, namely ε_1. It should be solved for a large number of refractive indexes $n(\lambda)$ in order to get the law $\varepsilon_1(\lambda)$. However, the equation (3.5) cannot be solved algebraically, but only numerically.

The method proposed to get the motion law $\varepsilon_1(\lambda)$ comprises the following steps:

- choice for a dispersive sort of glass within the visible range
- computation of refractive indexes for a large number of spectral lines, using the Sellmeier formula [5]:

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - B_2} + \frac{C_1\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3} + \ldots \tag{3.6}$$

- iterative calculus of deviation angle for different incidence angles around the one corresponding to minimum deviation
- interpolation of different types on the string of values (ε_1, λ) in order to get the best analytical expression for the motion law.

4. NUMERICAL APPLICATION

For the numerical application, the following input data were used:

- active angle of the prism: $\alpha = 60^o$
- choice for glass: P-LAF37 from the Schott Catalogue 2012 [5]. The coefficients in the interpolation formula (3.6) are:
  $B_1 = 1.76003244$
  $C_2 = 0.206332561$
  $B_2 = 1.39708831$
  $C_2 = 0.011658267$
  $B_3 = 0.0582087757$
  $C_3 = 130.748028$
- calculus of the refractive indexes within the range of (380...780) nm, step $\Delta\lambda$=5 nm.
  The numerical procedure needed the following steps:
- computation of the minimum deviation using a middle refractive index. The result was $\delta = 61.11^o$
- iterative calculus of $\varepsilon_1(\lambda)$ corresponding to the imposed value of δ. The results are presented in table 1.

The errors of δ are smaller than 0.5%.

The values in table 1 were used to draw graphs. Considering fulfilled the condition δ=constant, the pairs (λ, ε_1) were used to trace the variation of the incidence angle in respect with λ, which is deviated on the direction of the exit slit.

The graphs allow interpolation polynomial of different degree.

The best interpolation expression is a polynomial 6. It is represented in figure 3. The analytical expression which describes the function $\varepsilon_1(\lambda)$ is:

$$\varepsilon_1 = -12945\lambda^6 + 47473\lambda^5 - 71979\lambda^4 + 57751\lambda^3 - 25864\lambda^2 + 61349\lambda - 5955, \tag{4.1}$$

with a correlation factor R = 0.997.

However, it is very difficult to achieve practically.

A much realistic approach should be linearization.

Table 1. Corresponding values of λ, ε_1 and δ

ID	λ[mm]	n [-]	ε_1 [deg]	δ [deg]	ID	λ[mm]	n [-]	ε_1 [deg]	δ [deg]
1	0.380	1.792002	60.35	61.14	41	0.580	1.756154	75.35	61.06
2	0.385	1.790261	63.85	61.11	42	0.585	1.755719	75.40	61.06
3	0.390	1.788600	65.25	61.11	43	0.590	1.755294	75.47	61.07
4	0.395	1.787013	66.30	61.12	44	0.595	1.754880	75.58	61.07
5	0.400	1.785497	67.00	61.09	45	0.600	1.754477	75.67	61.08
6	0.405	1.784046	67.70	61.09	46	0.605	1.754083	75.75	61.09
7	0.410	1.782657	68.40	61.12	47	0.610	1.753699	75.84	61.10
8	0.415	1.781326	68.75	61.06	48	0.615	1.753324	75.93	61.11
9	0.420	1.780050	69.45	61.14	49	0.620	1.752957	76.02	61.13
10	0.425	1.778825	69.80	61.11	50	0.625	1.752600	76.10	61.14
11	0.430	1.777648	70.15	61.09	51	0.630	1.752250	76.19	61.15
12	0.435	1.776518	70.50	61.09	52	0.635	1.751908	76.28	61.17
13	0.440	1.775430	70.85	61.10	53	0.640	1.751575	76.33	61.14
14	0.445	1.774384	71.20	61.11	54	0.645	1.751248	76.38	61.10
15	0.450	1.773377	71.55	61.14	55	0.650	1.750928	76.42	61.12
16	0.455	1.772406	71.90	61.19	56	0.655	1.750616	76.45	61.14
17	0.460	1.771470	72.08	61.15	57	0.660	1.750310	76.45	61.11
18	0.465	1.770568	72.25	61.12	58	0.665	1.750011	76.45	61.08
19	0.470	1.769697	72.43	61.10	59	0.670	1.749717	76.54	61.10
20	0.475	1.768856	72.60	61.08	60	0.675	1.749430	76.63	61.12
21	0.480	1.768043	72.87	61.11	61	0.680	1.749149	76.63	61.09
22	0.485	1.767258	73.13	61.15	62	0.685	1.748873	76.63	61.06
23	0.490	1.766498	73.31	61.15	63	0.690	1.748603	76.72	61.09
24	0.495	1.765763	73.48	61.15	64	0.695	1.748338	76.80	61.12
25	0.500	1.765052	73.57	61.11	65	0.700	1.748079	76.80	61.09
26	0.505	1.764363	73.65	61.07	66	0.705	1.747824	76.80	61.06
27	0.510	1.763695	73.65	61.11	67	0.710	1.747574	76.89	61.09
28	0.515	1.763048	73.65	61.16	68	0.715	1.747329	76.98	61.13
29	0.520	1.762421	73.74	61.12	69	0.720	1.747088	76.98	61.10
30	0.525	1.761813	73.83	61.09	70	0.725	1.746851	76.98	61.08
31	0.530	1.761222	74.01	61.11	71	0.730	1.746619	77.07	61.11
32	0.535	1.760648	74.18	61.13	72	0.735	1.746391	77.15	61.14
33	0.540	1.760091	74.27	61.11	73	0.740	1.746167	77.15	61.12
34	0.545	1.759550	74.35	61.09	74	0.745	1.745947	77.15	61.09
35	0.550	1.759024	74.38	61.12	75	0.750	1.745731	77.24	61.13
36	0.555	1.758513	74.40	61.15	76	0.755	1.745518	77.33	61.16
37	0.560	1.758015	74.73	61.11	77	0.760	1.745309	77.33	61.14
38	0.565	1.757531	75.05	61.06	78	0.765	1.745103	77.33	61.12
39	0.570	1.757060	75.14	61.06	79	0.770	1.744901	77.33	61.10
40	0.575	1.756601	75.23	61.06	80	0.775	1.744702	77.33	61.08
					81	0.780	1.744506	77.33	61.06

Still, linearization is not possible along the entire wavelengths range at a convenient correlation factor. By more iterations, the linearization was achieved on three sections:

- (390...440) nm: $\varepsilon_1 = 108.1\lambda + 23.66$, R = 0.985
- (440...540) nm: $\varepsilon_1 = 31.42\lambda + 57.56$, R = 0.969
- (540...780) nm: $\varepsilon_1 = 12.15\lambda + 68.23$, R = 0.965.

Figure 4 illustrates the interpolation through linear segments.

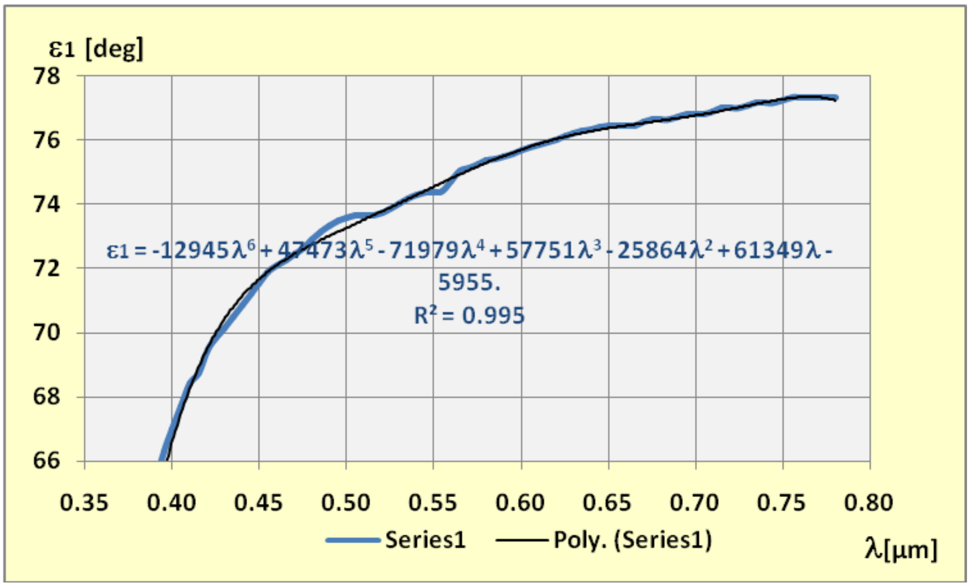

Figure 3 Interpolation through a polynomial 6

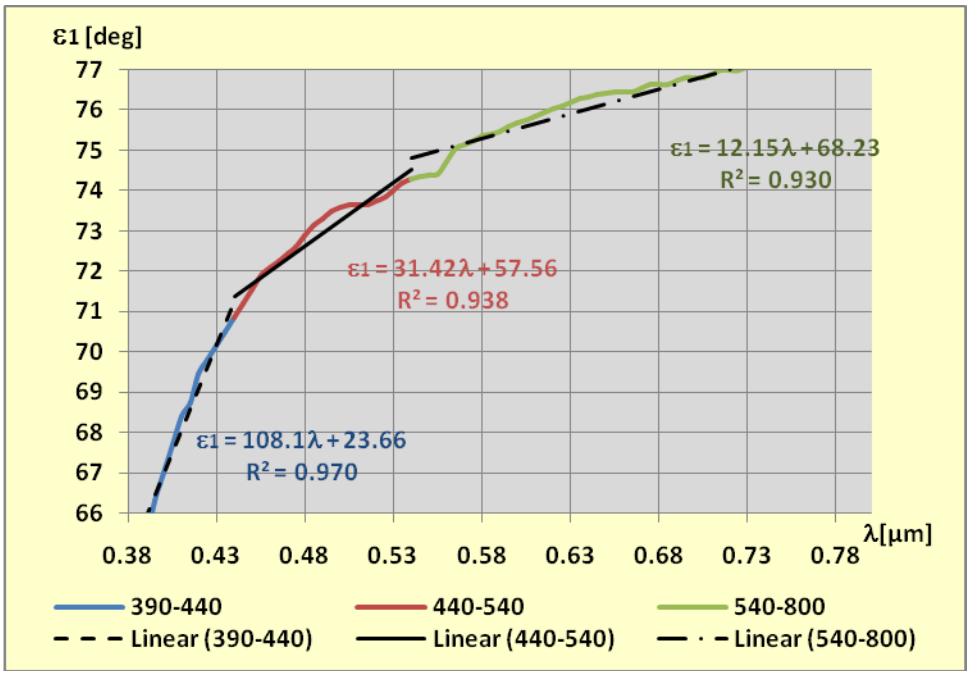

Figure 4 Interpolation through lines on three sections

5. CONCLUSIONS

As the entrance and exit slits of a spectral apparatus feature fixed positions, the dispersive prism needs to rotate so that radiations of different wavelengths are focused on the exit slit one by one.

Rotation of prism changes the incidence angle and thus the value of all angles involved in prism equations. The principle adopted in spectral apparatus construction is to establish a constant value of an angle, namely the deviation one. Usually the minimum deviation angle is chosen for starting the dimensioning of the prism.

The expression $\delta(\varepsilon_1)$ provides a transcendent equation, which is algebraically solvable. Numerical methods and iterative automated computation is needed.

The variation of the incidence angle ε_1 in respect with the wavelength is totally nonlinear. It is known that the spectrum provided by a prism within the visible range is nonlinear. The blue lines are much closer than the red ones.

For a given sort of glass and a constant value of deviation angle, a string of related values (λ, ε_1) was computed.

The graphic representation suggested a polynomial to interpolate the motion law of the prism. However, as practically such motion laws are inconvenient, linearization on three sections was preferred. The sections are far from equal as consequence of natural nonlinearity of the spectrum. The shortest section is the first one, corresponding to the blue region, very dense in spectral lines, whereas the longest section is the third one, corresponding to the yellow – red region, where spectral lines are wide.

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