Determination of the Reactions in Linkages with AutoLisp Functions

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Abstract—The present paper completes a previous paper in which were determined the positions, velocities and accelerations of a jointed quadrilateral mechanism with the aid of some certain AutoLisp functions. The completion consists in the realization of two AutoLisp functions which determine the reactions in the RRR dyad, at the active element and the animation of the vector polygonals or the animation of a wished reaction. The AutoLisp functions are easy to write, the relation that stay at the base of the thinking are vector relations used in the graph-analytical analysis. In the end are shown, as diagrams, the obtained numerical results.

Keywords—animation, AutoCAD, AutoLisp, reactions, mechanism

I. INTRODUCTION

In the design activity a remarkable aid is due to AutoLisp. This is a programming language that permits to the user the realization of mathematic calculation and the work with objects in AutoCAD.

In this paper we have the goal to solve the problem of kineto-static analysis with AutoLisp functions.

First of all we have to perform the kinematic analysis. In references [7], [8] we presented modalities of solving the problems of kinematic analysis with the aid of certain AutoLisp functions.

These are easy to be written, the used relations are the vector ones used in the graph-analytical analysis.

One obtains numerical results with the same precision of the assisted analytical methods.

To complete the reference [8] with the determination of the reactions in the kinematic joints of an articulated quadrilateral mechanism, we will write two AutoLisp functions.

The first one will determine the reactions in RRR dyad, while the second one determines the reactions at the active element.

The relations that stay at the base of thinking for the two functions are vector relations.

Finally, we will present the way of animation of the vector polygonals of forces.

II. DETERMINATION OF THE REACTIONS BY CLASSICAL GRAPH-ANALYTICAL METHOD

Dyads enter the category of the passive modular groups. These are statically determined and for this reason the kineto-static analysis of a mechanism starts from the last modular element, which is in reverse sense comparing to the kinematic analysis.

To realize an AutoLisp function used for any configuration of mechanism with rotational linkages, the elements of the dyad will have plan-parallel motion. Such a RRR dyad may be obtained from the bi-mobile mechanism \( O_1ABCO_2 \) in fig.1. Knowing the motions of elements 1 and 4, the mechanism \( O_1ABCO_2 \) is a desmodrome one.

In Fig. 1 is presented the isolation of the RRR dyad. One considers the elements of the dyad acted by known forces (given and inertial).

The element 2 is considered to be composed from the bars \( AB \) and \( BM \) solidary linked one to another and making the angle \( \alpha \).

Denoting by \( c_{21} \) and \( c_{22} \), respectively, the centers of gravity of the two bars, it is easier to determine the inertial forces of the element 2; these forces will be denoted by \( F_{21} \) and \( F_{22} \), respectively.

The inertial torques are denoted by \( M_{21} \) and \( M_{22} \), respectively.

Fig. 1. The RRR dyad isolated from the bi-mobile mechanism \( O_1ABCO_2 \)

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On the element 2 acts also the force $F$, with the expression (1):

$$ F = \begin{cases} 
0 & \text{for } \varphi \in [0,80^\circ) \\
50 \text{ N} & \text{for } \varphi \in [80^\circ,280^\circ] \\
0 & \text{for } \varphi \in [280^\circ,360^\circ] 
\end{cases} $$

(1)

The bars $AB$ and $BM$ are also acted by the weight forces $G_{21}$, and $G_{22}$ (the masses of the elements $m_{21} = 0.125$ kg, and $m_{22} = 0.125$ kg are known).

The element 3 has the center of gravity denoted by $c_3$, the mass $m_3$ ($m_3 = 0.125$ kg) and the weight $G_3$. The inertial force was denoted by $F_{i3}$, while the inertial torque was denoted by $M_{i3}$.

We considered that the bars are homogeneous; hence the centers of gravity are situated at the middle of the bars.

The torsor of the inertial forces is given by the relations:

$$ F_i = -m_i \vec{a}_i, \quad M_i = -J_i \vec{\omega}_i, $$

(2)

where $i$ takes the values 21, 22 and 3.

In figure the inertial forces ($\vec{F}_i$) were drawn in opposite sense to the accelerations of the elements, while $\vec{M}_i$ were represented in opposite direction to the angular accelerations.

That is why in the bottom part of Fig. 1 we represented the acceleration polygonals at which we added the accelerations of the gravity centers of the elements.

To determine the reactions in the kinematic joints $A$, $B$ and $C$, one isolates the elements, and decomposes the reactions in $A$ and $C$ into the normal components $R_{\alpha}^A$, $R_{\beta}^C$, and into the components along the bars $R_{\alpha}^B$, $R_{\beta}^B$.

From the equations of moments relative to the point $B$, one determines the components $R_{\alpha}^A$ and $R_{\beta}^B$. Thus, for the dyad in Fig. 1 it results

$$ R_{\alpha}^A = \left( -G_3 d_1 + F_{i3} d_2 - G_{21} d_3 + F_{i2} d_4 \
- F d_3 - M_{i21} - M_{i22} \right) / AB, $$

(3)

$$ R_{\alpha}^C = \frac{G_3 d_6 - F_{i3} d_7 - M_{i3}}{BC} $$

(4)

where the distances $d_1$, ..., $d_7$ correspond to the forces marked in Fig. 1.

Further on, writing the equilibrium of the forces for the all dyad,

$$ \overrightarrow{R}_{\alpha} + \overrightarrow{F}_{i1} + \overrightarrow{G}_{i1} + \overrightarrow{R}_{\beta} = 0, $$

(5)

result the components $R_{\alpha}^A$ and $R_{\beta}^B$; hence the reactions at $A$ and $C$ were completely determined,

$$ \overrightarrow{R}_{\alpha} = \overrightarrow{R}_{\alpha}^A + \overrightarrow{R}_{\alpha}^C, \quad \overrightarrow{R}_{\beta} = \overrightarrow{R}_{\beta}^A + \overrightarrow{R}_{\beta}^C. $$

(6)

From the equilibrium of forces which act upon the element 2,

$$ \overrightarrow{R}_{\alpha} + \overrightarrow{G}_{21} + \overrightarrow{F}_{i21} + \overrightarrow{G}_{22} + \overrightarrow{F}_{i22} + \overrightarrow{F} + \overrightarrow{R}_{B} = 0, $$

(7)

one determines the reaction $\overrightarrow{R}_{B}$.

The active element 1 has the mass $m_1$ ($m_1 = 0.05$ kg) and it is acted by: the weight $G_1$, the torsor of the inertial forces calculated relative to the rotational axis, and the reaction $\overrightarrow{R}_{A}$ (equal and of opposite direction to that calculated at the RRR dyad).

Because the active element has constant angular speed, the torsor of the inertial forces reduces at the inertial force $\overrightarrow{F}_{i1} = -m_1 \vec{a}_i$, where $c_1$ marks the center of gravity for the element 1.

Since $OA$ is a homogeneous bar, $c_1$ is at the middle of the element.

To determine the reactions of the active element, one isolates the element 1 (Fig. 2).

$$ \overrightarrow{M}_{c1} = G_1 d_8 - R_A d_9. $$

(9)
III. AUTOLISP FUNCTION TO DETERMINE THE REACTIONS OF THE RRR D\\n
Based on the relations (1) – (7) we wrote the AutoLisp function named "Reacţiuni_RRR", the content of which is as follows:

(Defun Reactiuni_RRR ()
 (setq m21 0.125)
 m22 0.125
 m3 0.125
 G21(* m21 9.8)
 G22(* m22 9.8)
 G3(* m3 9.8)
 J21/(* m21 1AB 1AB) 12)
 J22/(* m22 1BM 1BM) 12)
 J3/(* 1BC 1BC) 12) fmic(List 50 50))
 (Setq pctC21(Polar aprim (Angle aprim bprim) (/ (Distance aprim bprim) 2)))
 pctC22(Polar bprim (Angle bprim pctmprim) (/ (Distance bprim pctmprim) 2))
 pctC33(Polar bprim (Angle bprim cprim) (/ (Distance bprim cprim) 2)))
 (Setq ff121(* m21 acc12)
 ff122(* m22 acc22)
 ff13(* m3 acc3)
 Mi21(* J21 eps2 semnEps2)
 Mi22(* J22 eps2 semnEps2)
 Mi3(* J3 eps3 semnEps3)
 Fmare 0.0)
 (If (> fi1 70) (Setq Fmare 50))
 (If (> fi1 70) (Setq Fmare 50))
 (Setq ri21(- (/ pi 2) fi2)
 aRi21(- (Angle pctC21 jmic) fi2)
 aRi22(- alfaRad fi2 (/ pi 2))
 aRi22(- (Angle pctC22 jmic) (+ fi2 pi (* -1 alfaRad)))))
 aG3(- fi3 (* Pi 1.5))
 aF3(- fi3 (Angle pctC3 jmic))
 d1(* (/ 1AB 2) (Sin aG21))
 d2(* (/ 1AB 2) (Sin aF21))
 d3(* (/ 1BM 2) (Sin aG22))
 d4(* (/ 1BM 2) (Sin aF22))
 d5(* (/ 1BM (Sin aG22))
 d6(* (/ 1BC 2) (Sin aG3))
 d7(* (/ 1BC 2) (Sin aF3))
 (Setq RAn/ (+ (* G12 d1 -1) (* F121 d2)
 (* G22 d3 -1) (* F122 d4) (* Fmare d5 -1)
 (+ Mi21 Mi22 -1) 1AB)
 Rcn/ (+ (* G3 d6) (* F1i3 d7 -1) (* Mi3 -1)) 1BC)
 (Setq aRAn(+ fi2 (* pi 1.5))
 aRcn(- fi3 (* pi 1.5))
 pRAn(Polar fmic aRAn RAn)
 pG21(Polar aRAn (* pi 1.5) G21)
 pF121(Polar pg21 (Angle pctC21 jmic)
 fF121)
 pG22(Polar pF121 (* pi 1.5) G22)
 pF122(Polar pG22 (Angle pctC22 jmic)
 fF122)
 pFmare(Polar pF122 (* pi 1.5) Fmare)
 pG3(Polar pFmare (* pi 1.5) G3)
 pF33(Polar pG3 (Angle pctC3 jmic) fF33)
 pRcn(Polar pF33 aRcn RCN)
 pFrcPolar pRcn fi3 10)

The AutoLisp function starts by assigning values (Setq) to the masses of elements, continues with the calculation of gravity forces and inertial moments. It also assigns to the forces pole the coordinates \( f(50,50) \).

To determine the accelerations of the gravity centers \( a_{21}, a_{22} \) and \( a_{3} \), one firstly determine the coordinates of the middles of segments \( a'b', b'c' \) and \( j'm' \), respectively, from the accelerations polygonal presented in [5] with the AutoLisp function Polar.

The polygonal uses the accelerations determined with the AutoLisp function " Acceleratii " and presented in [5], and the names of the points in the accelerations plan. In the AutoLisp function the points were denoted by pctC21, pctC22 pctC33, the accelerations being the distances between these points (Distance) and the pole \( j \) of the accelerations.

The notations for these accelerations are \( acc21, acc22 \) and \( acc3 \), respectively.

The inertial forces given by (2) were denoted by \( ff121, ff121, ff13 \), and the inertial torques by \( Mi21, Mi22, Mi3 \). Due to the fact that on the elements the inertial forces draw in the opposite sense to accelerations, and the inertial moments in the opposite sense of the angular accelerations, we have to solve the problem of the sense of angular acceleration. The theory states that the geometric figure in the accelerations plan is similar to that in the position plan and rotated by the angle

\[
\gamma = \pi - \arctan \frac{\omega}{2} \quad (10)
\]

in the sense of the angular acceleration. To do this, one writes an AutoLisp function which compares the two angles from the accelerations and positions plans, respectively, the angle being measured from the end of the element in trigonometric sense. If the difference is equal to \( \gamma \), then \( semn \) takes the value 1, otherwise it takes the value –1.
To define the force $F$ given by (1), one used two interrogations $\mathbf{IF}$.

The determination of the distances $d_j$ from a point $B_j$ to the support straight line of the force $F_j$ (Fig. 3) is performed with the aid of the relation

$$d_j = C_j B_j \sin(\varphi_F - \varphi_j).$$  \hspace{1cm} (11)

We denoted by $\varphi_F$ the angle made by the force $F_j$ with the horizontal, and by $\varphi_j$ the angle made by the orientated segment $C_j B_j$ with the horizontal.

![Fig. 3. The distance from a point to a straight line.](image)

In the AutoLisp function, the distances were denoted by $d_1, \ldots, d_7$, they being determinate according to the representation in Fig. 3. Further on, were determined the components $R_A^1$ and $R_C^2$ with the algebraic relations (3) and (4).

To obtain the vector contour given by (5) one started Polar from the pole of forces $f$ with the angle $\varphi_2 + \frac{3\pi}{2}$ and the distance $R_A^1$.

One obtained the point $pRAn$. Further on, again with the function Polar, one obtains, one after another, the points $pG1$, $pFi21$, $pG22$, $pFi22$, $pF$, $pG3$, $pFi3$, $pRCn$, keeping into account the forces and their magnitudes, according to the vector polygonal [5].

To obtain the intersection between the parallel to $BC$ and passing through $pRCn$, and the parallel to $AB$ and passing through the pole $f$, one constructs two points $pFRCt$ and $pFRA$, situated on the two straight lines at an arbitrary distance (10).

With the function Inters one analyzes the four point $pRCn$, $pFRCt$, $fmic$ and $pFRA$ and if there exists point of intersection (Nil), then it is denoted by int.

We thus obtain the components: $R_A^1$ as distance (Distance) between the points $fmic$ and int, and $R_C^2$ as distance (Distance) between the points $pRCn$ and int. $R_A^1$, according to (6), is the distance between the points int and $pRAn$, while $R_C^2$ is the distance between the points $pFi3$ and int.

The reaction $R_O^2$, according to the vector relation (7), is obtained from the same contour as distance (Distance) between the points $pF$ and int.

We thus obtained the magnitudes of the reactions in the joints of RRR dyad, with no representation of the vector contour.

One may also draw the vector contour of the polygon using the AutoCAD command LINE.

This vector contour completes with arrows with the AutoLisp function "Sageti" described in [8] (Fig. 4).

![Fig. 4. The RRR dyad – the forces polygonal.](image)

**IV. AUTOLISP FUNCTION TO DETERMINE THE REACTIONS AT THE ACTIVE ELEMENT**

To obtain the components at the active element we write a new AutoLisp function named "React_Motor". It calls the function "React_Motor" or from the main function "R_3R" presented in [8]. Its construction follows the algorithm given by (8) and (9).

```lisp
(Defun React_Motor ()

  (Setq m1 0.05)
  (Setq G1 (* m1 9.8))
  (Setq pctC1 (Polar jmic (Angle jmic aprim)
                   (/ (Distance jmic aprim) 2)))

  (Setq pRAN (Polar int (Angle jmic pctC1)
                      pF1Rad))
  (Setq pG1 (Polar pF1 (* pi 1.5) G1))
  (Setq pRAN (Distance pG1 pRAN))
  (Setq sRA (* - (Angle pRAN int) (+ f1RAD (/ pi 2)))

  d8 (* / 10A (lOA (Sin f1RAD))
       d9 (* lOA (Sin aRA))
       Me1 (* G1 d8) (* RA d9)
  )
)
```

After the determination of the inertial force, one obtains $R_O^2$ using the previous vector contour. One uses a new pole of forces, the point $pRAn$. The force $R_A^2$ is at the point int. We continue from this point with the force $F_{i1}$ (it results the point $pF1$) and with the force $G_{i}$ (it results the point $pG1$). One thus obtained the reaction $R_O^2$ as distance (Distance) between the points $pG1$ and $pRAn$ (Fig. 5).
When one produces an imagine of the mechanism or its animation, the obtained results give a distinct note to the work comparing to the tables of the values obtained with the aid of the analytical methods.

With the aid of the AutoLisp functions, one obtains numerical results from vector equations, without any representation of the vector polygonal. But it is very useful and easy to give a representation of it.

To realize the animation, in fact one establishes the parameters of interest and how must be realized the representation. In the AutoLisp functions presented in the paper are determined all the parameters; the only thing we have to establish is what we wish visualize.

The application was realized for a jointed Chebyshev type quadrilateral mechanism for 360 positions of the active element. In reference [8] were presented the all six AutoLisp functions that realize the kinematic analysis of the mechanism. From the main function presented there, one calls the two functions presented in this paper, the first determines the reactions in the RRR dyad, and the second one the reactions at the active element.

In the function "R_3R" presented in [8] after the calling of the function "Reactiuni_RRR", the results may be also obtained in files of data. Based on these files one plots the graphics in Figs. 6–8, containing the variation of the reactions $R_A$, $R_B$ and $R_C$ as functions of the angle $\phi_1$ of the active element 1.

VI. CONCLUSIONS

The way of approaching of the kineto-static analysis in the paper hast at its basis the graphic methods which were forgotten a period of time because their low precision in obtaining the results.

With the apparition of the CAD soft, the notion of imprecision for the graphical methods disappeared. The results obtained with these softs have the same precision to that of the assisted analytical methods.

The vector mode in which works AutoCAD creates a strong closing to the vector methods used in mechanics and in the kinematic and kineto-static analysis of the mechanisms.

The present paper wishes, first of all, to prove this statement and to re-bring in nowadays the old graphic methods which are also intuitive.
One can combine the results obtained in the kinematic analysis with those from the kineto-static analysis.
For instance, one can realize the animation only for shaft 2 in Fig. 1, accompanied by the representation of the reactions $R_A$ and $R_B$ in the joints in $A$ and $B$. For this one must write the following lines of program:

```
(Command "Line" Amare Bmare ")
(Seq pFAmare(Polar Amare (Angle int pFAn) (* RA kf)))
(Command "Line" Amare pFAmare ")
(Seq pFBmare(Polar Bmare (Angle pF int) (* RB kf)))
(Command "Line" Bmare pFBmare ")
```

The following things were realized:
- the drawing of the segment $AB$ with the command Line,
- the determination of the point $pFAmare$ starting from the point $A$, Polar with the angle of the force $R_A$ and a length $RAk_f$, where by $k_f$ was denoted the scale of representation of the force in the forces polygonal,
- the drawing of the force $R_A$ at the point $A$ with the command,
- the determination and the drawing of the force $R_B$ at the point $B$ in a similar mode.

Further on, one chooses a window of visualization that is kept constant when the element 1 takes the 360 positions.
One can proceed in an analogous way for any other element or vector which accompany the representation in a certain animation.

**REFERENCES**