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ON OSCILATING SOLUTIONS OF DIFFERENTIAL EQUATIONS OF THE THIRD ORDER y′′′-|a(x)|·y=0

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Abstract—In this work, the given equation in which is a continuous function which meets Lipschitz's condition, has been solved by application of iteration sequence method. We have separated monotonous and exponential solutions, and then determine conditions under which the equation has oscillating solutions. We have also shown that type of the solution of the linear differential equation depends on the fore-sign of the coefficient, which means that it does not depend on its size, i.e. absolute value.

Keywords—Differential equations, successive approximations, iterations, Euler's exponential function, oscillations, Sturm's zeros.

I. Introduction

IN this work, an iteration approach is suggested for solving the equation

$$y''' - |a(x)| \cdot y = 0 \tag{1}$$

where a(x) is continuous coefficient. It is known thought by Liouville that, in general, differential equations with continuous coefficients define only exponential functions in broad sense. Above all, this include Euler's function e^x, but also different variants of exponentiality which depend on coefficient and order of equation and which will be written as

$$e^{\pm x}_{n;a_i(x)} \tag{2}$$

where n is order of the equation, while $a_i(x)$ are coefficients.

In even broader sense, there are also exponentials of the functions $e^{i\alpha x}$ which allow for oscillating solutions of $\{\cos \alpha x, \sin \alpha x\}$ type and also many generalizations – non-elementary exponent, sine and cosine. In this way, every separate linear differential equation can define its separate exponents and its separate trigonometries

$$Tg\{\sin_{n;a_i(x)} x; \cos_{n;a_i(x)} x\} \tag{3}$$

the basis of which are contained in the classic Euclid's trigonometry.

Therefore, generality regarding the type of solutions of linear homogenous differential equation of the n[th] order $L_n(y) = 0$ is not so big. Thus, the Goursat's principle is confirmed: "Give me the singularities and I will tell you the solutions."

To be more concrete, in this work we easily differentiate monotonous and exponential solutions of the equation (1) in opposition to $y''' + |a(x)| \cdot y = 0$ where alternations of signs of the iteration members does not produce clear picture on the type of solutions. Furthermore, we also give oscillating solutions of the above equation. The previous inductive idea is confirmed in the sense that, regarding properties and quality of solutions of linear equation, the fore-sign of coefficients is more important than their value – absolute value.

II.. Main Results

2.1. Iteration sequence method

Let us have linear equation:

$$y''' - |a(x)| \cdot y = 0 \tag{4}$$

in which *a(x)* is continuous coefficient. Then from the normal form of the equation

$$y''' = |a(x)| \cdot y \tag{5}$$

which fulfills Lipschitz's condition, it follows that all iterations will have the same sign, so there will be no problems with alternating sequences. If we write the first integrals:

$$\begin{cases} y'' = C_1 + \int |a(x)| \cdot y(x)dx \\ y' = C_1 x + C_2 + \iint |a(x)| \cdot y(x)dx^2 \\ y = C_1 \cdot \dfrac{x^2}{2} + C_2 x + C_3 + \iiint |a(x)| \cdot y(x)dx^3 \end{cases} \tag{6}$$

we can define iteration sequence

$$y^{[n]}(x) = C_1 \cdot \frac{x^2}{2} + C_2 x + C_3 + \iiint |a(x)| \cdot y^{[n-1]}(x)dx^3 \quad n=1,2,3,\ldots \tag{7}$$

In order to make it simpler and avoid big summations, we choose constants C_i: (C_1, C_2, C_3): (0,0,1), (0,1,0) and (1,0,0) so we obtain three integral equations for three particular integrals y_1, y_2, y_3. Each of these equations is solved by iteration sequence method.

2.2. The first exponential solution

By choosing constants C_i *(i=1,2,3)* so that *(C_1, C_2, C_3)= (0,0,1)*, we obtain particular solution:

$$y_1 = 1 + \iiint |a(x)| \cdot y_1(x)dx^3 . \tag{8}$$

For integral equation (8) we define iteration sequence:

$$y_1^{[n]}(x) = 1 + \iiint |a(x)| \cdot y_1^{[n-1]}(x)dx^3, \quad n=1,2,3,\ldots \tag{9}$$

with initial approximation $y_1^{[0]}(x) = 1$.

Based on recurrent formulae (9), we have the following members of iteration sequence:

$$y_1^{[1]} = 1 + \iiint a(x)y_1^{[0]}(x)dx^3 = 1 + \iiint a(x)dx^3$$

$$y_1^{[2]} = 1 + \iiint a(x)y^{[1]}(x)dx^3 = 1 + \iiint a(x)\left[1 + \iiint a(x)dx^3\right]dx^3 =$$

$$= 1 + \iiint a(x)dx^3 + \iiint a(x)dx^3 \iiint a(x)dx^3$$

$$y_1^{[3]} = 1 + \iiint a(x)y^{[2]}(x)dx^3 = 1 + \iiint a(x)dx^3\left[1 + \iiint a(x)dx^3 + \iiint a(x)dx^3 \iiint a(x)dx^3\right] =$$

$$= 1 + \iiint a + \iiint a \iiint a + \iiint a \iiint a \iiint a$$

Consequently, without any difficulties, by means of mathematical induction we find:

$$y_1^{[n]} = 1 + \iiint a + \iiint a \iiint a + \iiint a \iiint a \iiint a + \ldots + \iiint a \ldots \iiint a$$

or shortly written

$$y_1^{[n]} = 1 + \sum_{k=1}^{n} \iiint a \ldots \iiint a \cdot dx^3 , \tag{10}$$

where we put *a(x)>0* instead of $|a(x)|$.

Since *a(x)* is positive continuous function which fulfills Lipschitz's condition, it is easy to prove that integral operator (9) is contracting, so the sequence of successive approximations converges towards solution of the equation (4). If we take that the limits of the integral are *[0,x]* and then that interval expand to the interval $[0, x+\Delta x]$, then we have:

$$\int_0^{x+\Delta x} \iint a(x)dx^3 = \int_0^{x} \iint a(x)dx^3 + \int_x^{x+\Delta x} \iint a(x)dx^3$$

And since it is obvious that $\int_x^{x+\Delta x} \iint a(x)dx^3 > 0$ and the same is valid for all addends from (10), it follows that:

$$y_1^{[n]}(x+\Delta x) > y_1^{[n]}(x) \tag{11}$$

which means that iterations monotonously rise because their limit function i.e. the solution $y_1(x)$ of the differential equation for the initial conditions *(0,0,1)* is monotonous. Therefore the solution:

$$y_1(x) = \lim_{n\to\infty} y_1^{[n]} = 1 + \sum_{n=1}^{\infty} \iiint a \ldots \iiint a \tag{12}$$

is some monotonously increasing exponential function.

2.3. The second exponential solution

If we take the initial conditions *(0,1,0)* for constants C_i we obtain particular solution:

$$y_2 = x + \iiint a(x)y_2(x)dx^3 \tag{13}$$

For integral equation, which defines particular integral y_2, the recursive formula defines iteration sequence.

$$y_2^{[n]} = x + \iiint a(x)y_2^{[n-1]}(x)dx^3 , \quad n=1,2,3,\ldots,$$

with initial approximation $y_2^{[0]}(x) = x$. Therefore, we obtain:

$$y_2^{[1]} = x + \iiint xa(x)dx^3$$

$$y_2^{[2]} = x + \iiint a(x)y_2^{[1]}(x)dx^3 = x + \iiint a(x)[x + \iiint xa(x)dx^3]dx^3 =$$

$$= x + \iiint xa(x)dx^3 + \iiint a(x)dx^3 \iiint xa(x)dx^3$$

$$y_2^{[3]} = x + \iiint xa(x)dx^3 + \iiint a(x) \iiint xa(x)dx^3 + \iiint a \iiint a \iiint xa(x)dx^3$$

and so on. By induction we obtain:

$$y_2^{[n]} = x + \sum_{k=1}^{n} \iiint a \iiint a \ldots \iiint a \iiint xa$$

for *x>0* . Since *a(x)>0* is completely analogical, we also obtain:

$$y_2(x) = \lim_{n\to\infty} y_2^{[n]}(x) \tag{14}$$

which is monotonous and positive solution and it can be confirmed by first derivative. Apart from this, it is obvious that $y_2(x) > y_1(x)$ for $x>1$. Therefore the solutions are worked out by exponentials.

2.4. The third exponential solutions

For initial conditions *(1,0,0)*, we obtain the particular integral:

$$y_3 = \frac{x^2}{2} + \iiint a(x) y_3(x) dx^3 \tag{15}$$

for which iteration sequence is defined by following formula:

$$y_3^{[n]} = \frac{x^2}{2} + \iiint a(x) y_3^{[n-1]}(x) dx^3 \text{, } n=1,2,3,..., \tag{16}$$

with initial approximation $y_3^{[0]}(x) = \frac{x^2}{2}$.

The members of the sequence $\{y_3^{[n]}\}$ are as follows:

$$y_3^{[1]} = \frac{x^2}{2} + \iiint a(x) y_3^{[0]}(x) dx^3 = \frac{x^2}{2} + \iiint \frac{x^2}{2} a(x) dx^3$$

$$y_3^{[2]} = \frac{x^2}{2} + \iiint a(x) y_3^{[1]}(x) dx^3 = \frac{x^2}{2} + \iiint a(x)[\frac{x^2}{2} + \iiint \frac{x^2}{x} a(x) dx^3] dx^3 =$$

$$= \frac{x^2}{2} + \iiint \frac{x^2}{2} a(x) dx^3 + \iiint a(x) dx^3 \iiint \frac{x^2}{2} a(x) dx^3$$

$$y_3^{[3]} = \frac{x^2}{2} + \iiint a(x) y_3^{[2]}(x) dx^3 =$$

$$= \frac{x^2}{2} + \iiint \frac{x^2}{2} a(x) dx^3 + \iiint a(x) dx^3 \iiint \frac{x^2}{2} a(x) dx^3 + \iiint a(x) dx^3 \iiint a(x) dx^3 \iiint \frac{x^2}{2} a(x) dx^3$$

and so on . By mathematical induction, with no difficulties, we obtain the following:

$$y_3^{[n]} = \frac{x^2}{2} + \sum_{k=1}^{n} \iiint a \iiint a .... \iiint a \iiint \frac{x^2}{2} a(x) dx^3$$

Since the integral operator is contraction, then the iteration sequence $\{y_3^{[n]}\}$, *n=1,2,3,...,* is convergent and its limit value is the following solution:

$$y_3(x) = \lim_{n \to \infty} y_3^{[n]}(x) \tag{17}$$

which is positive and monotonously increasing function where for *x>1* the following inequality is valid $y_3 > y_2 > y_1$. From the derivatives $y_i' > 0$, $y_i'' > 0$ and *i=1,2,3*, we can make conclusion about monotony, concavity and mutual supremacy, i.e. their ratio of increase.

For the Wronskian $W(x) = W(y_1, y_2, y_3)$ which in this case is the third order determinant, after longer calculation, we can conclude that it is different from zero. It means that y_1, y_2, y_3 are linearly independent particular solutions of the given equation. Therefore, the general solution of the equation (4) is as follows:

$$\begin{aligned} y(x) = {} & C_1[1 + \int_3 a + \int_3 a \int_3 a + \int_3 a \int_3 a \int_3 a + ...] + \\ & + C_2[x + \int_3 xa + \int_3 a \int_3 xa + \int_3 a \int_3 a \int_3 xa + ...] + \\ & + C_3[\frac{x^2}{2} + \int_3 \frac{x^2}{2} a + \int_3 a \int_3 \frac{x^2}{2} a + \int_3 a \int_3 a \int_3 \frac{x^2}{2} a + ...] \end{aligned} \tag{18}$$

If all the constants C_i, *i=1,2 and 3* are positive and general, the solution *y(x)* is positive, as well, and also monotonous. Therefore, for the above mentioned choice $C_i > 0$ there are different exponential solutions. However, according to the classical theory of homogenous linear differential equations, the equation (4) can have also oscillating solutions because it is of the odd order.

Theorem 1. If all the constants C_i,(*i=1,2,3)* are positive, then all solutions of the equation (4) are monotonous.

Theorem 2. Apart from monotonous solutions, the equation (4) has also oscillating solutions if at least one coefficient in (18) is negative.

2.5. Euler's exponential function

If the constants *Ci* are chosen in such a way that $(C_1, C_2, C_3)=(1,1,1)$, then we obtain particular solution of the equation (4) of the following form:

$$\begin{aligned} y_1(x) = {} & 1 + \int_3 a + \int_3 a \int_3 a + \int_3 a \int_3 a \int_3 a + ... + \\ & + \frac{x}{1!} + \int_3 xa + \int_3 a \int_3 xa + \int_3 a \int_3 a \int_3 xa + ... + \\ & + \frac{x^2}{2} + \int_3 \frac{x^2}{2} a + \int_3 a \int_3 \frac{x^2}{2} a + \int_3 a \int_3 a \int_3 \frac{x^2}{2} a + ... \end{aligned} \tag{19}$$

In order to show that an exponential function is hidden here, let us first take that *a=const.* In that case, it follows that:

$$y_1(x) = 1 + x + \frac{x^2}{2} + \frac{ax^3}{3!} + \frac{ax^4}{4!} + \frac{ax^5}{5} + \frac{a^2x^6}{6!} + \frac{a^2x^7}{7!} + \frac{a^2x^8}{8!} + \frac{a^3x^9}{9!} + \frac{a^3x^{10}}{10!} + \frac{a^3x^{11}}{11!} + ...$$

Especially for *a=0*, we obtain trivial solution $y = 1 + x + \frac{x^2}{2}$ of the differential equation $y''' = 0$, while for $a \equiv 1$, obviously there is function e^x. This function is solution of the equation $y''' = y$ which is obtained from iteration sequence (19) when we substitute *a* with one, because then:

$$y_1(x) = 1 + \frac{x^3}{3!} + \frac{x^6}{6!} + \frac{x^9}{9!} + ... + x + \frac{x^4}{4!} + \frac{x^7}{7!} + \frac{x^{10}}{10!} + ... + x^2 + \frac{x^5}{5!} + \frac{x^8}{8!} + \frac{x^{11}}{11!} + ...$$

If we organize differently the sum and group the members respecting the increasing exponents, we obtain:

$$y_1(x) = (1 + \frac{x}{1!} + \frac{x^2}{2!}) + (\frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}) + (\frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!}) + ... +$$
$$+ (\frac{x^n}{n!} + \frac{x^{n+1}}{(n+1)!} + \frac{x^{n+2}}{(n+2)!}) + ... = e^x$$

Finally, for positive and substantially high *a(x)* from the majorant $|a(x)| < A$, it is obvious that the solution $y_1(x)$ is majorated by exponential function e^{Ax}.
Therefore, for particular solution $y_1(x)$ given with (19), we have the right to introduce the following definition.

Definition. Particular solution (11) of the equation (1) given with

$$y_1(x) = 1 + x + \frac{x^2}{2!} + \int_3 (1 + x + \frac{x^2}{2!})a + \int_3 a \int_3 (1 + x + \frac{x^2}{2!})a + \int_3 a \int_3 a \int_3 (1 + x\frac{x^2}{2!})a + ...$$

is general monotonous increasing exponential function of the third order with the basis *a(x)* which is marked with $e^x_{III,a(x)}$.

Nevertheless, since there are another two exponential functions, because the equation (4) can have three exponential solutions, by their linear combination, we can define general hyperbolic functions of the third order with basis *a(x)*.

$$ch_{III,a(x)}x, \; sh_{III,a(x)}x, \; th_{III,a(x)}x, \tag{20}$$

However, this is not the goal of this work.

2.6. Decreasing the order. Oscillating solutions.

If we manage to separate one monotonous solution y_1 of the equation (4), then, based on general theory of differential equations, this equation can have oscillating solutions for every other choice of integral constants. In this case, as we know, by making substitution $y = y_1 \cdot Z$, the equation of the third order (4) can be derived into the second order respecting the new unknown function $Z = Z(x)$:

$$(y_1''' \cdot Z + 3y_1'' \cdot Z' + 3y_1' \cdot Z'' + y_1 \cdot Z''') - a(x) \cdot y_1 \cdot Z = 0$$

By grouping members, we obtain

$$Z \cdot (y_1''' - a(x) \cdot y_1) + (3y_1'' \cdot Z' + 3y_1' \cdot Z'' + y_1 \cdot Z''') = 0$$

Since y_1 is particular integral, it remains:

$$Z''' + 3 \cdot \frac{y_1'}{y_1} \cdot Z'' + 3\frac{y_1''}{y_1} \cdot Z' = 0 \tag{21}$$

This equation, by substitution,

$$Z' = W; \; W = W(x) \tag{22}$$

is derived into the equation of the second order:

$$W'' + 3 \cdot \frac{y_1'}{y_1} \cdot W' + 3\frac{y_1''}{y_1} \cdot W = 0 \tag{23}$$

It is important here that y_1 is exponential function, so it has no zeros, which means that coefficients of the equations (21) and (23) are continuous. Apart from this, based on general theory, it follows that these can have oscillating solutions. The equation (23) is derived to the canonical form by introducing substitution.

$$W = U \cdot e^{-\frac{1}{2}\int \frac{3y_1'}{y_1}dx} = U \cdot e^{-\frac{3}{2}\ln y_1} = \frac{U}{\sqrt{y_1^3}} \tag{24}$$

So that canonical form should be as follows:

$$U'' + \Phi(x) \cdot U = 0 \tag{25}$$

where $\Phi(x)$, according of the basic theory of differential equations of the second order is:

$$\Phi(x) = \frac{3y_1''}{y_1} - \frac{1}{2}(\frac{3y_1'}{y_1})' - \frac{1}{4}(\frac{3y_1'}{y_1})^2,$$

i.e.

$$\Phi(x) = \frac{3}{2}(\frac{y_1''}{y_1} - \frac{1}{2} \cdot \frac{y_1'^2}{y_1^2}) \tag{26}$$

The equation (4) has oscillating solutions if continuous coefficient $\Phi(x)$ is positive on canonical interval *[0,x]*. Also, in order to have infinite number of oscillations on the semi axis, $[0,+\infty]$ must be big enough ($\int_0^{+\infty} \Phi(x)dx$ diverges), so that it could trigger the oscillations. Therefore, for $\Phi(x) > 0$, the equation (25) has oscillating solutions.

$$U_1 = \cos_{\Phi(x)} x, \; U_2 = \sin_{\Phi(x)} x. \tag{27}$$

Therefore, from (24) it follows:

$$W_1 = \frac{1}{\sqrt{y_1^3}} \cdot \cos_{\Phi(x)} x, \; W_2 = \frac{1}{\sqrt{y_1^3}} \cdot \sin_{\Phi(x)} x. \tag{28}$$

And from (22) it follows that $Z = \int W(x)dx$, which leads to:

$$Z_1 = A + \int \frac{1}{\sqrt{y_1^3}} \cdot \cos_{\Phi(x)} xdx;$$

$$Z_2 = B + \int \frac{1}{\sqrt{y_1^3}} \cdot \sin_{\Phi(x)} xdx;$$

(29)

Thus, the substitution $y = y_1 \cdot Z$ leads to:

$$y_2 = e^x_{III,a(x)} \int \frac{1}{\sqrt{e^{3x}_{III,a(x)}}} \cdot \cos_{\Phi(x)} xdx;$$

$$y_3 = e^x_{III,a(x)} \int \frac{1}{\sqrt{e^{3x}_{III,a(x)}}} \cdot \sin_{\Phi(x)} xdx;$$

(30)

Theorem 3. General solution of the equation (4) is linear combination which consists of one linear and two oscillating solutions:

$$y = C_1 e^x_{III,a(x)} + e^x_{III,a(x)} [C_2 \int e^{-\frac{3}{2}x}_{III,a(x)} \cdot \cos_{\Phi(x)} xdx + C_3 \int$$

where $e^x_{III,a(x)}$ is generated exponential function of the third order, while $\sin_{\Phi(x)} x$, $\cos_{\Phi(x)} x$ are general since and cosine of the second order with basis $\Phi(x)$ given in the form of functional sequences.

$$\cos_{\Phi(x)} x = 1 - \iint \Phi + \iint \Phi \iint \Phi - \iint \Phi \iint \Phi \iint \Phi + ...$$

(31)

$$\sin_{\Phi(x)} x = x - \iint x\Phi + \iint \Phi \iint x\Phi - \iint \Phi \iint \Phi \iint x\Phi + ...$$

(32)

which are determined by iteration sequences method. The basis $\Phi(x)$ is given with (26) and it depends on y_1, and consequently on coefficient $|a(x)| : \Phi = \Phi(|a(x)|)$.

<u>*Example 1:*</u> Let us have elementary differential equation with constant coefficients:

$$y''' - y = 0$$

By applying method of the characteristic equation, if we put $y = e^{rx}$, we obtain:

$$r^3 e^{rx} - e^{rx} = 0 \Rightarrow r^3 = 1 \Rightarrow r = \sqrt[3]{1} \Rightarrow$$

$$r = \cos\frac{2k\pi}{3} + i\sin\frac{2k\pi}{3}, \; k=0,1,2$$

Therefore, we obtain:

$$r = 1$$

$$r_2 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = \frac{-1 + i\sqrt{3}}{2}$$

$$r_3 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = \frac{-1 - i\sqrt{3}}{2}$$

So, the particular integrals are as follows:

$$y_1 = e^x, \; y_2 = e^{-\frac{x}{2}} \cos\frac{\sqrt{3}}{2}x, \; y_3 = e^{-\frac{x}{2}} \sin\frac{\sqrt{3}}{2}x,$$

and general solution should be:

$$y = C_1 e^x + e^{-\frac{x}{2}} (C_2 \cos\frac{\sqrt{3}}{2}x + C_3 \sin\frac{\sqrt{3}}{2}x)$$

This solution is dominantly exponential, for higher *x*.

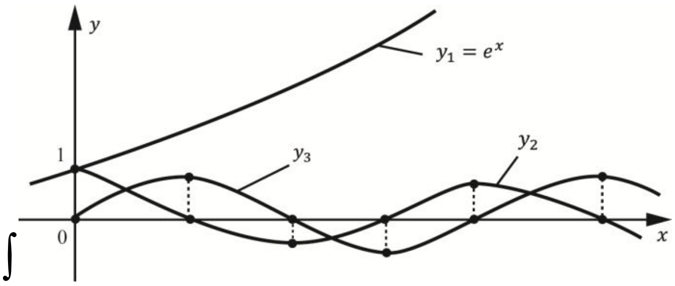

Fig. 1. Particular solutions

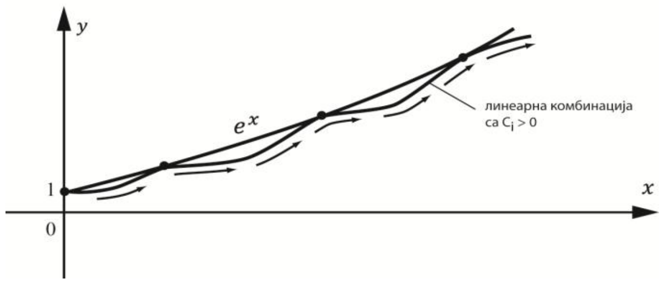

Fig. 2. General solutions

However, the monotonous solution is still possible, but with changeable curve if $C_2 \cos\frac{\sqrt{3}}{2}x + C_1 \sin\frac{\sqrt{3}}{2}x < 0$ i.e. due to equation of the second order. If $y'' = 0$, then from $y'' > 0$ (concavity) it passes into $y'' < 0$ (convexity). The following is important:

<u>Note 1.</u> General solution, i.e. any solution does not depend only on particular integrals y_1, y_2 and y_3, but it also considerably depends on integrating constants

C_1, C_2 and C_3. Thus, in this case, combination (18) consisting of positive exponential function, if some constant C_i is lower than zero, it becomes oscillating. Therefore, here, we again encounter the problem and properties of the sum of exponents.

Note 2. The equation (4) is not canonical, but special. The canonical equation of the third order should be $y''' + ay' + by = 0$ and it depends on two coefficients. However, this work is a good basis and indication for some broader procedure for multi-member equations, where iterations are more complicated.

Note 3. Now it is not difficult to approach the problem of solving Sturm's zeros for the equation (4) or for similar equation $y''' - |a(x)| \cdot y = 0$. (See [1]). In this way we can solve dilemmas which exist in literature whether Sturm's theorems can be applied for the equations of higher order.

Note 4. There is no reason for not believing that such procedure will not be valid for differential equations of the n^{th} order, as well, only with adequate technical difficulties (complexity of iterations which makes making conclusions more difficult).

III. Conclusion

Non-canonical equation of the third order has been solved by iteration sequence method, which differs from the classic Sturm's method. By applying this method, we have determined three linearly independent particular integrals y1, y2 and y3. We have defined general exponential function with the basis a(x), which we have marked. Then, by combining exponential functions, we have introduced hyperbolic functions of the third order. We have shown that given equation can have all three monotonous solutions, which can have maximum one zero. If, according to Sturm, that zero exists, it can have one monotonous and two oscillating solutions [4-5, 9-11].

Oscillating solutions have been determined by reducing the given equation (in case when one its particular integral is known) to completely linear homogenous differential equation of the second order, which subsequently has been reduced to canonic equation. The canonic equation of the second order has been solved by iteration method and determined its oscillating solutions, number of zero oscillations and locations of zero oscillations (see [1-3, 6-8]).

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