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ESTIMATION OF ANGULAR VELOCITY SUBSEQUENT TO A PLANE IMPACT

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Abstract**—**The paper presents the methodology for evaluating the velocity of a ball following a plane impact. The estimative angular velocity value is required when the energy balance is made. The present method provides an approximate angular velocity but adequate to answer the question whether the angular velocity value influences significantly or not, the energy balance equation of impact process.

Keywords**—**angular velocity, plane collision, energy balance, experimental collisions

I. Introduction

PHENOMEN modeling in the perspective of impact mechanics can be made involving more or less complex methods, considering the aimed goal. In the case when the variation of certain parameters during impact-considered as taking place in a finite interval of time, is not required, the study is straightforward as the variation of different parameters is described by certain coefficients expressing the ratio between the values of the parameter in attention, after and before collision. The coefficient of restitution is perhaps the most important among these parameters.

This coefficient can be defined in two ways [1], [2]: kinematical (Newton) using the normal components of velocities after and before impact and dynamical, (Poisson) [3], assuming a process taking place in a finite time period, during which two phases are identified, namely compression and restitution. In the later case, the coefficient of restitution is described by the ratio of percussions from restitution phase and compression phase, respectively. It is interesting to emphasise that, this parameter is required regardless of the adopted study method.

A reference work is due to Lankarani, [4] who studies the centric impact of two metallic spheres and models the process as a process with internal friction. In his work, Lankarani [4] considers that the kinetic energy variation of the system is recovered as heat obtained by internal friction. Moreover, it is accepted the hypothesis that the work, for the two phases, compression and restitution, is the same. Based on this assumption, the damping coefficient is found by applying the energy balance equation.

The hypothesis of equality between friction work for the two phases of the impact leads to a model that can be applied only for materials exhibiting quasi-elastic behaviour (model for values of coefficient of restitution with $e>0.9$).

The Lankarani model was adjusted by Flores, [5], who left the assumption of identical internal friction work for the two impact phases and supposing that in the phases' plane the characteristic point describes an ellipse, similar to Kelvin-Voigt model [6], finds another value for the damping coefficient and thus extends the model applicability for the entire materials range since the coefficient of restitution can get any value $e\in(0,1)$.

Another model was proposed also by Lankarani, [7], for the case of elastoplastic impact of two balls, assuming that the entire kinetic energy variation is restored as work of plastic deformation. In the models mentioned the balls were approximated as punctiform bodies and the problem of angular velocity did not arise.

The problem becomes intricate when the colliding bodies are no longer considered as punctiform bodies.

In this case, describing the velocity distribution requires knowing the velocity of a point of the rigid v_0 and the angular velocity of the rigid, ω**,** [8].

Another more complex impact model refers to the bidimensional impact. There are several methods of approaching this situation but the method proposed by Routh, [9], the plane of percussions method, is notable.

Wang and Mason, [10], show that for the plane impact of two bodies, the use of kinematical and dynamical coefficients, respectively, for this type of problems, leads to different results. The only situation when the results are identical is when the bodies are considered punctiform. Additionally, in the attempt to classify the plane frictional impact, they reach the hypothesis that tangential percussions can exist independently of the normal ones, fact that is impossible to accept for usual forces.

II. THEORETICAL CONSIDERATIONS

For a mechanical system, generally, in vector mechanics, [11], three theorems are presented: the principle of momentum, the principle of moment of momentum and the principle of kinetic energy. The effect of internal forces from the system can be estimated only by using the last of these theorems.

$$\mathbf{E'_c = E''_c + L_{ext} + L_{am} + L_{pl}} \qquad (1)$$

where E', E" represents the energy of the system before and after collision, respectively, L_{ext} is the work energy of external forces, L_{am} is the work of internal friction forces (damping work) recovered as heat, and L_{pl} is the work lost by plastic deformation process. The kinetic energy of a body in plane-parallel motion is, [8]:

$$\mathbf{E_c = \frac{1}{2} m v_C^2 + \frac{1}{2} J_z \omega^2} \qquad (2)$$

The simplest bi-dimensional impact problems are the cases when a ball hits a motionless body. Due to spherical symmetry of the ball it is convenient to use it as colliding body, thus ensuring a higher probability for the reproducibility of events. In this case, the downside consists in the fact that the angular velocity cannot be estimated, due to the lack of a fixed mark. One solution is attaching a mark as fixed part, but if it interposes between the sphere and the immobile body, the result of the experiment could be altered. With the assumption that in impact processes the angular velocity is neglected, is easier to apply (2) and (1).

III. THE PRINCIPLE OF METHOD

An impact experiment is proposed, using a ball bearing. The ball is set in a free fall motion and collides an inclined plane and consequently gains an angular movement. As a consequence of the impact with the inclined plane, the mass centre of the ball describes a parabolic trajectory in a vertical plane and the ball also rotates about an horizontal axis. The angular velocity will be determined by breakdown in frames the recorded movie of the impact.

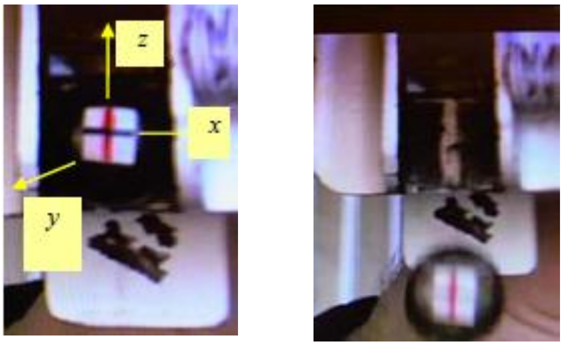

Fig. 1. The ball in the launcher and immediately after release

For testing that the ball doesn't rotate after the free fall launching, an attempt was made by sticking an adherent mark with axes traced on it, as shown in Fig. 1.

From Fig. 1 it can be observed that after releasing the ball, it doesn't revolve. Another fact that must be pointed out is that the dimension of the stain on the mark is too small to be observed. Therefore, the advantage of spherical symmetry was withdrawn and a new device was proposed, by attaching to the ball an aluminum rod, long enough to be observed after the collision with the immobile body. To connect the rod, a threaded whole was machined into the ball by electro erosion, Fig. 2.

Fig. 2. The ball and the attached rod

For the case of a short rod, the orientation of the rod was not enabled by movie decomposition into frames, Fig. 3.

In order to ensure a value of angular velocity as great as possible, a greater coefficient of friction is required and the collision was set with a rubber block, thus ensuring μ=0.7, [12].

Fig. 3. The ball with short rod before and after impact

The moment of inertia about the center of mass J_z was found using a CAD application, resulting $J_z=4.26 \cdot 10^{-5}$ ($kg \cdot m^2$). The computed mass is 0.082(kg) and the measured mass is 0.081(kg). The additional aluminum rod shifted the position of the centre of mass with 9.3(mm).

Only an estimation of the magnitude order for angular velocity is aimed and therefore the conditions for dynamic equivalence were not taken into consideration. For an accurate analysis, the mass of the ball, the centre of gravity and the inertia tensor should be the same as for the assembly used in experimental tests, the holed sphere and rod system.

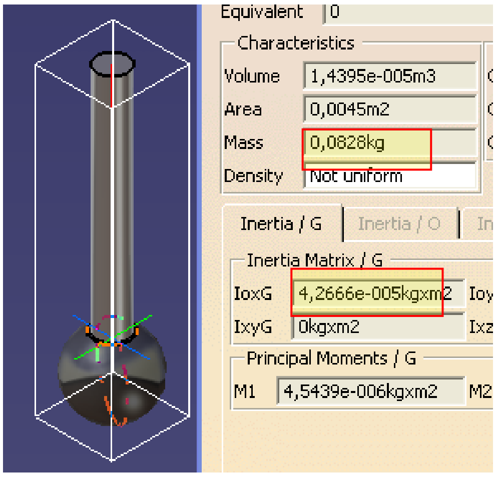

Fig. 4. Estimation of the moment of inertia about the centre of mass $\mathbf{J}_z$ using CAD software

After attaching the longer rod, at a movie recording velocity of 30(frame/s), the ruled surface described by the rod can observed.

The position of the rod can be observed from Fig. 5. The ruled surface described by the rod axis is explained by the time during which the circuit of the video camera color sensor is active reported to the time of acquisition between two successive images. For position estimation, the first straight line that impresses the image during the period of image acquisition must be considered as reference line.

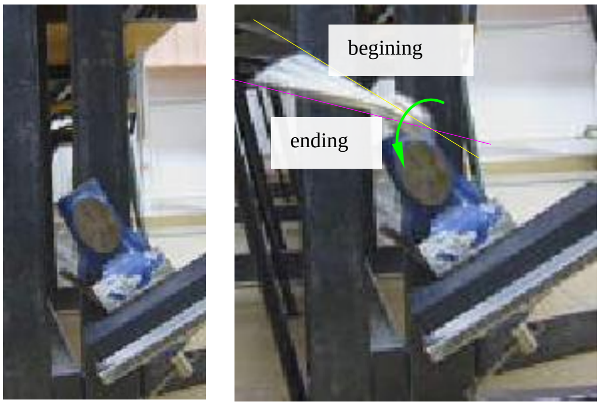

Fig. 5. The ball with the long rod before and after impact

The moment of momentum theorem about the center of mass has the form:

$$\frac{d\bar{\mathbf{K}}_C}{dt} = \bar{\mathbf{r}}_C \times \bar{\mathbf{G}} \qquad (3)$$

Because $\bar{\mathbf{r}}_c$ the position vector of the centre of mass about the centre of mass is:

$$\bar{\mathbf{r}}_C \equiv \mathbf{0} \qquad (4)$$

it results:

$$\mathbf{K}_C \equiv \mathbf{J}_z\omega = \mathbf{const.} \qquad (5)$$

Consequently, after collision, the angular velocity of the ball-rod assembly remains constant during the whole flight period. The orientation or the rod is précised from start by using the beginning reference line, since during impression the rod also has a translational motion that might affect the aspect of the ruled surface. A succession of images of frames from the recorded movie is considered for finding the angular velocity. On this images there are traced the lines from the beginning of impression, Fig. 6. After that, on the current image, a parallel to the line from previous image is traced and the angle between these lines is measured, Fig. 7. This angle represents the rotation related to the interval Δt=1/30(s), with the error introduced by the visioning angle.

Fig. 6. Estimation of rotation angle between two successive image frames: previous frame

Fig. 7. Estimation of rotation angle between two successive image frames: present frame

Applying the methodology for all the frames following the collision, with extra care in image manipulation (not to deform and alter the angles), the angular velocity of the ball-rod assembly can be found by printing the images and making the measurements. For the present work, there were used six images and there were obtained the following values of angular velocity: $\omega=\{13.6;11.5;13.6;\ 14.1;14.1;14.6\}$(rad/s). The average value of these angular velocities is expected to be close to the actual angular velocity: ω_{med}=13.58 (rad/s).

The velocity before collision is 2.33(m/s) and results that the initial kinetic energy is:

$$E'_C = \frac{1}{2}mv^2 = 0.233(J)$$

After collision, the rotational kinetic energy is:

$$E''_{Crot} = \frac{1}{2}J_z\omega^2 = 0.0039(J)$$

Thus, from the total initial energy of the ball-rod assembly only a percentage of 1.76% is retrieved as rotational kinetic energy.

As a first conclusion, for collision analysis, from engineering point of view, the effect of angular velocity can be neglected as a first approximation. This assumption leads to substantially simplified relations.

Regarding the angular velocity, the limit supposition that the whole initial kinetic energy of the ball is found as rotational energy after collision is made, in order to estimate the value of angular velocity of the ball.

$$\frac{1}{2}mv'^2 = \frac{1}{2}J_{zsph}\omega^2_{sph}$$

where

$$J_{zsph} = \frac{2}{5}mr^2$$

represents the axial momentum of the ball about the centre of mass and r is the ball radius. After computation, it is obtained $\omega_{sph} \cong 150$ (rad/s).

Compared to the experimental average angular velocity, this theoretical maximum value is more than ten times greater.

The present paper approaches a model of an actual experiment that provides additional information concerning the plane collision of a ball. After impact, the ball has both translational and rotational energy but the translational energy has a decisive proportion.

IV. CONCLUSIONS

Finding the post impact angular velocity of a body in plane collision is a difficult task. The most straightforward method of estimating the angular velocity is to assume that the entire translational kinetic energy variation is retrieved as rotational kinetic energy, thus neglecting the damping work or/and plastic deformation work.

The present paper tries a more adequate estimation. Attaching to the colliding body an aluminum rod, small enough as not to alter significantly the inertial characteristics, on the frames of the movie record of the motion after impact, the aluminum rod generates a set of ruled surfaces that allow for finding the average angular velocity.

For a real example, the angular velocity value is ten times smaller than under the assumption of complete transfer of translational energy into rotational energy. Moreover, the results show that for a first approximation, the dynamic effects of ball rotation can be neglected.

For more accurate results required by the necessity of internal friction estimation, a high speed video camera should be utilized. Therefore, the dimensions of the ruled surfaces from the frames captures will diminish. A supplementary condition concerns the dynamical equivalence between the ball and the system holed-sphere and rod. The presented methodology is an alternative for the methods where motion sensors are employed. The sensors necessary in such experiments are subjected to shock, together with the test-rig assembly and specialized sensors, protected from shocks and moreover, are quite expensive and must be periodically calibrated. Additionally, in order not the involve the system's dynamics, the sensors must be of wireless type. In consequence, the method proposed in the present paper is one of the less costly and straightforward, using minimal logistics.

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