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CALCULATION ALGORITHM FOR DETERMINING KINEMATIC PARAMETERS OF THE CARDAN JOINT MECHANISM WITH TECHNICAL (GEOMETRICAL) DEVIATIONS

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Abstract—The technological (geometrical) deviations of component elements of the cardan joint mechanism lead to the changine of kinematic parameters of the mechanism. For determining the influence of both angular and axis deviations, over the kinematic parameters of the cardan joint mechanism, it is necessary to consider the cardan joint, not as a spherical quadrilateral but as a particular case of RCCC mechanism where by C, R was noted the cylindrical kinematic pair respectively the rotation kinematic pair. In this paper is established the calculation algorithm for determining kinematic parameters of the cardan joint mechanism with technical (geometrical) deviations at component elements.

Keywords—cardan joint, geometrical deviations, kinematic pair.

I. INTRODUCTION

THE mechanism with one cardanic join [3], [5], [7]. it's an RRRR mechanism and a particular case of a spatial RCCC mechanism, where by C, R was noted the cylindrical kinematic rotation couple.
The technological deviations determine the apparition of some efforts in the intermediary couple of the cardanic joint. In order for one to have a measure for these displacement it is first necessary to study the RCCC spatial mechanism kinematics.

II. THE POSITIONAL ANALYSIS OF THE RCCC MECHANISM

The RCCC mechanism (see Fig. 1.) is made of four elements [4], [5], [7]. noted with 1, 2, 3 and 4, the forth element (the base) being fixed and the elements being connected through the kinematic couples O_1, O_2, O_3 and O_4, the O_1 being the rotation couple and O_2, O_3 and O_4 being the cylindrical kinematic pairs.

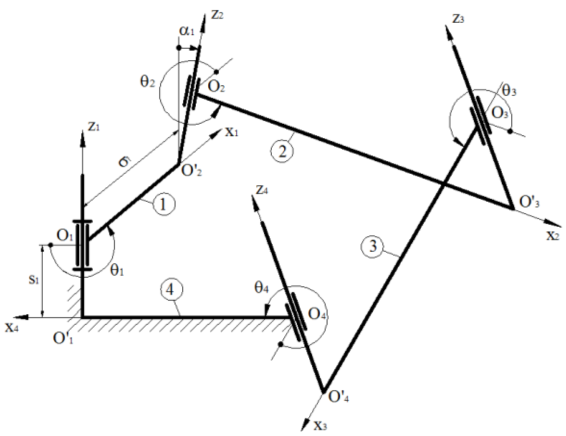

Fig. 1. RCCC Spatial Mechanism.

The axes of the kinematic pairs are noted with $O_i' z_i$, $i = 1,2,\ldots$, and the following perpendiculars are noted with $O_i' O_{i+1}'$, $i = 1,2,3,4$, point O_5 being identical with point O_1.

One notates with $\sigma_i, \alpha_i, i = 1,2,3,4$ the length of the axes and the angle between them. So it is chosen a local reference system $O_i x_i y_i z_i$, $i = 1,2,3,4$ so that the axes $O_i x_i$ to be situated on the shared perpendicular of the axes $O_i' z_i, O_{i+1}' z_{i+1}$.

It is noted with s_i the distances $O_i' O_i$ and with θ_i the angle between the axes $O_{i-1} x_{i-1}, O_i x_i, i = 1,2,3,4$.
In these conditions, the geometrical parameters s_i , σ_i , α_i , $i = 1,2,3,4$ being known, the positional analysis for determining $\theta_2, \theta_3, \theta_4, s_2, s_3, s_4$ is based on the angle θ_1.
From the equation of rotations closing, using the diagram „$\theta\alpha$" [6]. and the order 3,4,1 and 2 is obtained the following equation:

$$\mathbf{A_3(\theta_1)s\theta_4 - B_3(\theta_1)c\theta_4 + C_3(\theta_1) = 0} \quad (1)$$

where :

$$\begin{aligned}&\mathbf{A_3(\theta_1) = s\alpha_3 s\theta_1 s\alpha_1}\\&\mathbf{B_3(\theta_1) = s\alpha_3(c\alpha_4 c\theta_1 s\alpha_1 + s\alpha_4 c\alpha}\\&\mathbf{C_3(\theta_1) = -c\alpha_3 s\alpha_4 c\theta_1 s\alpha_1 + c\alpha_3 c\alpha_4 c\alpha_1 - c\alpha_2.}\end{aligned} \quad (2)$$

The trigonometrically functions *cos*, *sin* being noted with *c*, *s*. Through the conventional derivate D of the relations

$$\mathbf{D(c\theta_i) = -s_i s\theta_i ; D(s\theta_i) = s_i c\theta_i .} \quad (3)$$
$$\mathbf{D(c\alpha_i) = -\sigma_i s\alpha_i ; D(s\alpha_i) = \sigma_i c\alpha.} \quad (4)$$

is obtained the equation:

$$\mathbf{D_3 s_4 + F_3 s_1 + F_3\sigma_1 + G_3\sigma_2 + H_3\sigma_3 + K_3\sigma_4 = 0.} \quad (5)$$

The angle θ_4 is determined by solving the equation (1) and through the equation (5) is known the parameter s_4. With circular permutations the relations follows:

$$\mathbf{A_2(\theta_4)s\theta_3 - B_2(\theta_4)c\theta_3 + C_2(\theta_4) = 0.} \quad (6)$$
$$\mathbf{A_1(\theta_3)s\theta_2 - B_1(\theta_3)c\theta_2 + C_1(\theta_3) = 0.} \quad (7)$$

from where are determined, in order, the angles θ_3 and θ_2 and also the equations:

$$\mathbf{D_2 s_3 + E_2 s_4 + F_2\sigma_4 + G_2\sigma_1 + H_2\sigma_2 + K_2\sigma_3 = 0.} \quad (8)$$
$$\mathbf{D_1 s_2 + E_1 s_3 + F_1\sigma_3 + G_1\sigma_4 + H_1\sigma_1 + K_1\sigma_2 = 0.} \quad (9)$$

from which are determined the parameters s_3, s_2.
The expression of the coefficients $A_i, B_i, C_i, D_i, E_i, F_i, G_i, H_i, K_i$, i=3,2,1 ,are given in the TABLE 1. from the paper [2].
In the initial position , $\theta_i^0 = 0$ the expressions are obtained

$$\begin{aligned}&\mathbf{A_3 = 0}\\&\mathbf{B_3 = s\alpha_3 s(\alpha_1 + \alpha_4)}\\&\mathbf{C_3 = c\alpha_3 c(\alpha_1 + \alpha_4) - c\alpha_2.}\end{aligned} \quad (10)$$

and it results that:

$$\mathbf{c\theta_4^0 = \frac{c\alpha_3 c(\alpha_1 + \alpha_4) - c\alpha_2}{s\alpha_3 s(\alpha_1 + \alpha_4)}.} \quad (11)$$

For the correctly interpretation of the geometrical deviations it is first necessary to make some:

the perpendiculars common between the axes with the index *i* , *i+1* are noted with O_i, O'_{i+1};

the direction of the axis $O_i x_i$ is given by the rotation direction of the axis $O'_i z_i$ over the axis $O'_{i+1} z_{i+1}$, direction that also specifies the measurement direction of the angle α_i;

the positive measurement direction of angle θ_i between the axes $O_{i-1}x_{i-1}, O_i x_i$, is given by the direction of the $O_i x_i$ axis rotation around the axis $O'_i z_i$.

III. The Influence of Technological Deviations over the Kinematic Parameters

A kinematic diagram that represents a mechanism with one cardan joint, with all geometrical deviations possible [1]., is presented in Fig. 2.

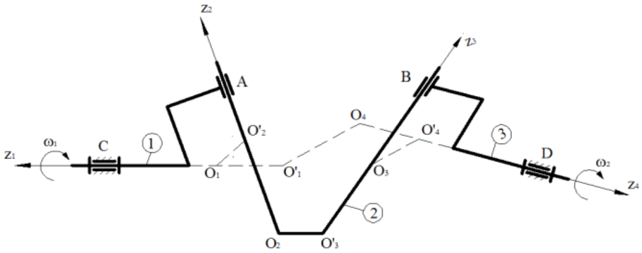

Fig. 2. Geometrical deviations.

These deviations are small and fulfill the condition:

$$\begin{aligned}&\mathbf{\alpha_i = \frac{\pi}{2} + \Delta\alpha_i , i = 1,2,3., \alpha_4 = \pi - \alpha}\\&\mathbf{\sigma_i = O_i O'_{i+1} ., i = 1,2., \sigma_4 = O_4 O'_i .}\end{aligned} \quad (12)$$

The angularly deviation of the main shaft bracket is defined by the parameter $\Delta\alpha_1$ and the smoothness deviations for the same bracket is given by the parameter σ_1.

The angularly deviation of the cardanic cross 2 is given by the parameter $\Delta\alpha_2$ and also the deviation from smoothness is given by the parameter σ_2.

The angularly deviation of the driven shaft bracket 3 is given by the parameter $\Delta\alpha_3$ and the smoothness deviation is given by the parameter σ_3.

The angularly deviation of the driven shaft 3 depending on the driving shaft 1 is given by the parameter σ_4.

As shown in default of shafts 1 and 3 are known the points (see Fig. 2.) $O_4, O'_1, O_1, O'_2, O_2, O'_3, O_3, O'_4$ are overlaid with point O (see Fig. 9.) and the kinematic cylindrical couples A , B and D (see Fig. 2.) become rotation kinematic couples (there are no displacements s_2, s_3, s_4 along the axes Oz_2, Oz_3, Oz_4).

The existence of technical deviations conducts to the displacements $s_i, i = 1,2,3,4$ and by blocking those the excess efforts from the rotation kinematic pairs of the cardan joint mechanism.

In order to determine these displacements it is first necessary to calculate the angularly parameters

$\theta_2, \theta_3, \theta_4$ variation depending on the angle θ_1 .

IV. THE DETERMINING PARAMETERS $\theta_2, \theta_3, \theta_4$

The angularly parameters $\theta_2, \theta_3, \theta_4$ is determined from the system of equations

$$\mathbf{A_i s\theta_{i+1} - B_1 c\theta_{i+1} + C_i = 0, i = 1,2,3.} \quad (13)$$

To this purpose, one uses the Newton method [9], [10]. and with the notations

$$[\theta] = \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}, [\Delta\theta] = \begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \\ \Delta\theta_4 \end{bmatrix}. \quad (14)$$

$$\mathbf{\Psi_i = A_i s\theta_{i+1} - B_i c\theta_{i+1} + C_i, i = 1,2,3.} \quad (15)$$

$$\{\Psi\} = \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \end{bmatrix}. \quad (16)$$

$$\begin{aligned} \mathbf{A_1^*} &= \mathbf{s\alpha_1 s\alpha_3 c\theta_3} \\ \mathbf{B_1^*} &= \mathbf{-s\alpha_1 c\alpha_2 s\theta_3 s\alpha_3} \\ \mathbf{C_1^*} &= \mathbf{c\alpha_1 s\alpha_2 s\theta_3 s\alpha_2}. \end{aligned} \quad (17)$$

$$\begin{aligned} \mathbf{A_2^*} &= \mathbf{s\alpha_2 s\alpha_4 c\theta_4} \\ \mathbf{B_2^*} &= \mathbf{-s\alpha_2 c\alpha_3 s\theta_4 s\alpha_4} \\ \mathbf{C_2^*} &= \mathbf{c\alpha_2 s\alpha_3 s\theta_4 s\alpha_4}. \end{aligned} \quad (18)$$

$$[\mathbf{J}] = \begin{bmatrix} \mathbf{A_1 c\theta_2 + B_1 s\theta_2} & \mathbf{A_1^* s\theta_2 - B_1^* c\theta_2 + C_1^*} & \mathbf{0} \\ \mathbf{0} & \mathbf{A_2 c\theta_3 + B_2 s\theta_3} & \mathbf{A_2^* s\theta_3 - B_2^* c\theta_3 + C_2^*} \\ \mathbf{0} & \mathbf{0} & \mathbf{A_3 c\theta_4 + B_3 s\theta_4} \end{bmatrix} \quad (19)$$

is obtained the matric equation

$$\Delta\theta = [\mathbf{J}]^{-1}\{\Psi\} \quad (20)$$

from which results the variation $\{\Delta\theta\}$ for the known values of angles $\theta_1, \theta_2, \theta_3, \theta_4$.

V. CALCULATION ALGORITHM

For the numerical calculation one uses the following algorithm:
The following parameters are considered as known:

$$\alpha_i = \frac{\pi}{2} + \Delta\alpha_i, i = 1,2,3, s_1, \sigma_j, j = 1,2,3,4, \alpha. \quad (21)$$

a. The precision of calculation is chosen

$$\varepsilon = 0{,}001 \mathrm{rad} \quad (22)$$

b. The step change of angle is chosen

$$\theta_1, \Delta\theta_1 = 1^\circ. \quad (23)$$

c. Is considered $\theta_1 = 0^\circ$ and the approximate value of the $\{\Delta\theta\}$ matrix according to those previously established

$$\{\Delta\theta\} = \left[\frac{\pi}{2}, \frac{3\pi}{2} + \alpha, \frac{\pi}{2}\right]^T. \quad (24)$$

d. The coefficients $A_i, B_i, C_i, i = 1,2,3.$ are calculated with the relations from the TABLE 1 from the paper [2].

e. The functions Ψ_i and the matrix $\{\Psi\}$ are calculated with the relations (15) and (16);

f. The coefficients $A_i^*, B_i^*, C_i^*, i = 1,2$ are calculated with the relations (17) and (18) and the jacobian $[J]$ with the relation (19) ;

g. The matrix $\{\Delta\theta\}$ are calculated with the relation (20);

h. The updated matrix is calculated

i.

$$\{\theta\} \to \{\theta\} + \{\Delta\theta\}. \quad (25)$$

j. If are fulfilling the condition

$$(\Delta\theta_i) < \varepsilon, i = 2,3,4. \quad (26)$$

the calculation is continued and if you are not satisfied is repeat from point d), for new values of the angles

$$\theta_i, i = 2,3,4. \quad (27)$$

k. θ_1 is replaced with

$$\theta_1 + \Delta\theta_1. \quad (28)$$

l. For $\{\Delta\theta\}$ is considered as approximate value, the value of the previous step and the calculations is repeat from the point d);

m. The TABLE I is made

TABLE I
KINEMATIC PARAMETERS

θ1 [degrees]	θ4 [degrees]	θ3 [degrees]	θ2 [degrees]	s4 [m]	s3 [m]	s2 [m]
0°	-	-	-	-	-	-
1°	-	-	-	-	-	-
3°	-	-	-	-	-	-
-	-	-	-	-	-	-
-	-	-	-	-	-	-
-	-	-	-	-	-	-
-	-	-	-	-	-	-
360°	-	-	-	-	-	-

where

$$s_4 = -\frac{1}{D_3}(E_3 s_1 + F_3\sigma_1 + G_3\sigma_2 + H_3\sigma_3 + K_3\sigma_4). \quad (29)$$

$$s_3 = -\frac{1}{D_2}(E_2 s_4 + F_2\sigma_4 + G_2\sigma_1 + H_2\sigma_2 + K_2\sigma_3). \quad (30)$$

$$\mathbf{s_2 = -\frac{1}{D_1}(E_1 s_3 + F_1\sigma_3 + G_1\sigma_4 + H_1\sigma_1 + K_1\sigma_2)}. \quad (31)$$

$E_i, F_i, G_i, H_i, K_i, i = 1,2,3.$ are calculated from the TABLE 1. from the paper [2].

n. Plot the graphics

$$\mathbf{\theta_i(\theta_1), s_i(\theta_1), i = 2,3,4.} \quad (32)$$

VI. Numerical Application

One considers a cardanic joint for which:

$$\mathbf{\alpha = 0^\circ, \Delta\alpha_2 = 0{,}001rad, \Delta\alpha_1 = \Delta\alpha_3 = 0} \\ \mathbf{s_1 = 0, \sigma_i = 0, i = 1,2,3,4.} \quad (33)$$

The variation graphs are presented in Fig. 3., Fig. 4., and Fig. 5.

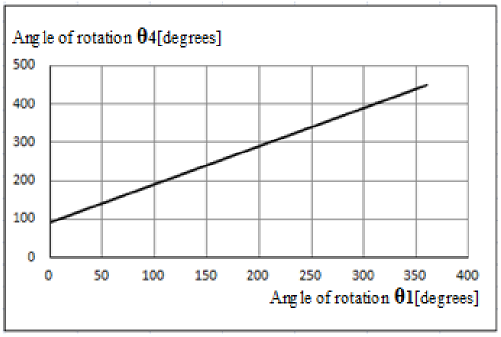

Fig. 3. The variation of rotation angle **θ4** for *α=0°*

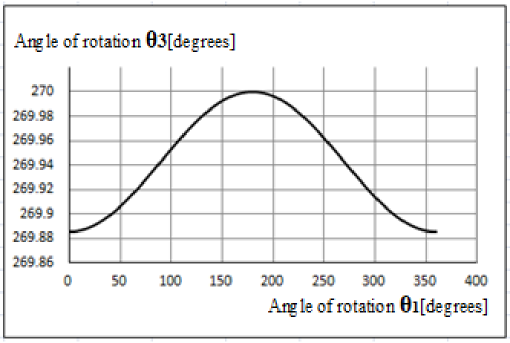

Fig. 4. The variation of rotation angle **θ3** for *α=0°*

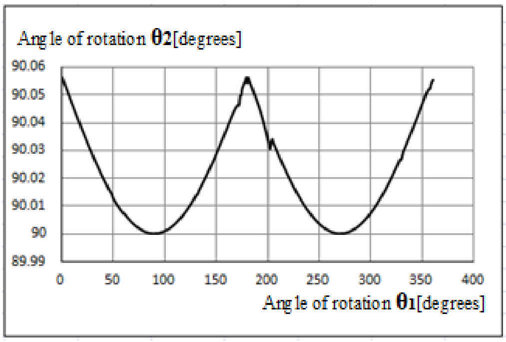

Fig. 5. The variation of rotation angle **θ2** for *α=0°*

VII. Conclusions

For the normal cardan join with technical deviations with $\alpha = 0^\circ$ and $\Delta\alpha_i = 0{,}001 rad$,when θ_1 covers the interval $0 - 360^\circ$ the angle θ_4 varies between $90 - 450^\circ$; the angle θ_3 varies between $269{,}88 - 270^\circ$; the angle θ_2 varies between $90 - 90{,}06^\circ$.

The influence of σ_i and s_1 deviations over the angles $\theta_4; \theta_3; \theta_2$ are insignificant in value.

The variation of angles $\Delta\alpha_i$ i=1,2,3 does no influence the displacements s_i , i=2,3,4.

The displacements s_i , i=2,3,4. are influenced only by the value of the σ_i and s_1 parameters.

For $\alpha = 0^\circ$, the variation curves form of the kinematic parameters are alike.

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