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THE TECHNICAL AND GEOMETRICAL DEVIATIONS OF CARDAN JOINT MECHANISM

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Abstract—During the manufacturing and montage process of cardan transmissions, at the component elements may inevitable appear technical (geometrical) deviations. These deviations lead to the changine of kinematic parameters of the mechanism.For the determination of these changes it is first necessary to identify these deviations and give them a geometrical interpretation.

By having as a purpose the identifying of both angular and axis deviations, over the kinematic parameters of the cardan joint mechanism, it is necessary to consider the cardan joint, not as a spherical quadrilateral but as a particular case of RCCC mechanism where by C, R was noted the cylindrical kinematic pair respectively the rotation kinematic pair.

Keywords—cardan joint, geometrical deviations, kinematic pair.

I. INTODUCTION

THE displacement laws of the cardan cross and of the driven fork of the cardan joint are deducted from the displacement equations of the rod and of the second crank, written for the 4R spherical quadrilateral mechanism **(**see Fig. 1.**).**

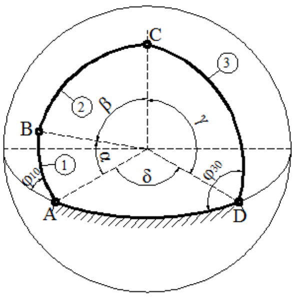

Fig. 1. 4R Spherical quadrilateral mechanism.

In these equations arises the angles $\alpha,\beta,\gamma,\delta$ (relation 9.24, page 143, [1].) and by particularization $\alpha=\beta=\gamma=\frac{\pi}{2}$ are deducted the displacement laws of the technically usable cardan joint elements.

The technical deviations that can be analyzed are the following [2]:

a) *non-orthogonally deviations of the cardan cross, in this case the angle β becomes equal with $\frac{\pi}{2}+\Delta\beta$;*

b) *non-orthogonal deviations of the parts of forks axes, case where angles α,γ also became equal with $\frac{\pi}{2}+\Delta\alpha, \frac{\pi}{2}+\Delta\gamma$.*

By having as a purpose the identifying of both angular and axis deviations, over kinematic of the cardan mechanism, it is necessary to consider the cardan joint, not as a spherical quadrilateral but as a particular case of RCCC mechanism where by C, R was noted the cylindrical kinematic pair respectively the rotation kinematic pair mechanism where the distances $\sigma_i, i=1,2,3,4$ between the successive axes of the kinematic pairs intervene.

II. THE RCCC SPATIAL MECHANISM

The RCCC mechanism (see Fig. 2.) is made of four elements [3], [4], [5]. noted with 1, 2, 3 and 4, the forth element (the base) being fixed and the elements being connected through the kinematic pairs O_1,O_2,O_3 and O_4, the O_1 being the rotation couple and O_2,O_3 and O_4 being the cylindrical kinematic pairs.

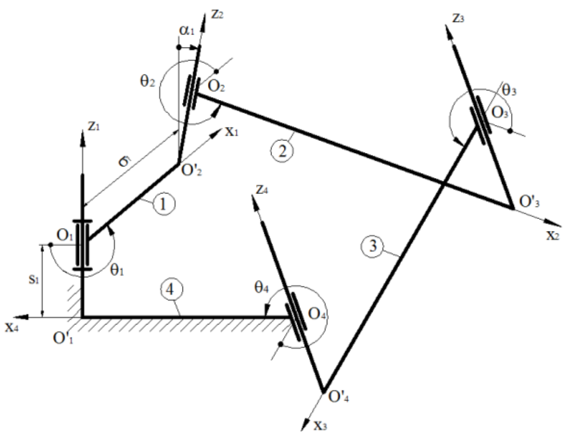

Fig. 2. RCCC Spatial Mechanism.

The axes of the kinematic pairs are noted with $O_i'z_i$, $i=1,2,\ldots$, and the following perpendiculars are noted with $O_i'O_{i+1}'$, $i=1,2,3,4$, point O_5 being identical with point O_1.

One notates with $\sigma_i, \alpha_i, i=1,2,3,4$ the length of the axes and the angle between them.So it is chosen a local reference system $O_i x_i y_i z_i$, $i=1,2,3,4$ so that the axes $O_i x_i$ to be situated on the shared perpendicular of the axes $O_i'z_i, O_{i+1}'z_{i+1}$.

It is noted with s_i the distances $O_i'O_i$ and with θ_i the angle between the axes $O_{i-1}x_{i-1}, O_i x_i, i=1,2,3,4$.

For the correctly interpretation of the geometrical deviations it is first necessary to make some:

1) the perpendiculars common between the axes with the index *i* , *i+1* are noted with O_i, O_{i+1}';
2) the direction of the axis $O_i x_i$ is given by the rotation direction of the axis $O_i'z_i$ over the axis $O_{i+1}'z_{i+1}$, direction that also specifies the measurement direction of the angle α_i;
3) the positive measurement direction of angle θ_i between the axes $O_{i-1}x_{i-1}, O_i x_i$, is given by the direction of the $O_i x_i$ axis rotation around the axis $O_i'z_i$.

III. THE NORMAL CARDAN JOINT

The cardanic joint enables the transmission of the rotation movement from the shaft 1to the shaft 3 through the cardanic cross 2.

The cardanic cross is tied to the forks of the shafts 1 and 3 through the cinematic rotation couples A, A' and also B, B' (see Fig. 3.).

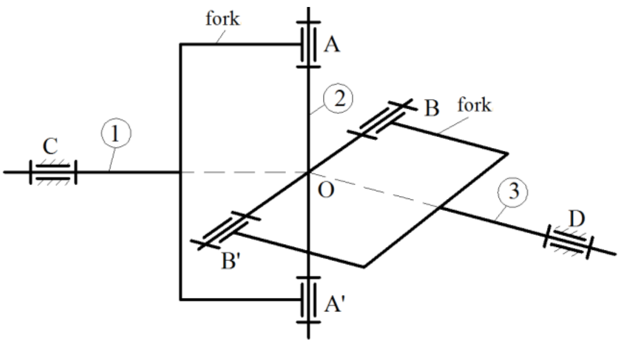

Fig. 3. Cardan joint mechanism.

The mechanism with one cardanic joint [1], [3], [6]. it's an RRRR mechanism and a particular case of a spatial RCCC mechanism, where by C, R [1]. was noted the cylindrical kinematic pair respectively the rotation kinematic pair.

In Fig. 4. is presented the equivalent mechanism with the mechanism from Fig. 1 where the bigger arc BC was replaced with a crank bar BOC.

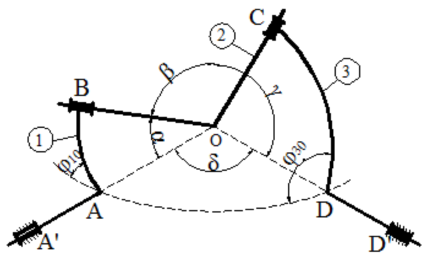

Fig. 4. 4R Equivalent spherical quadrilateral mechanism.

The simplest cardan cross joint is present in Fig. 5,

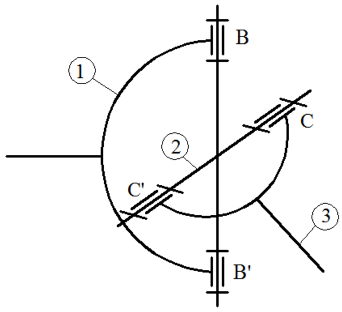

Fig. 5. The cardan cross joint.

where by relinquishing the passive rotation couples B', C' another kinematic scheme is obtained from Fig. 6, where the axis are concurrent in O.

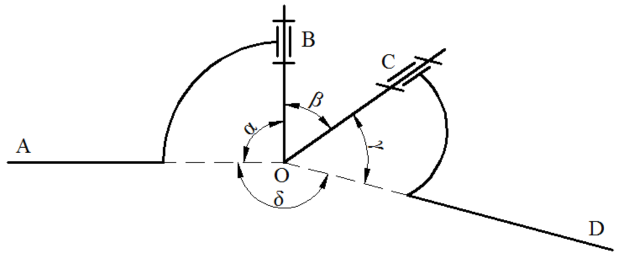

Fig. 6. The equivalent kinematic scheme.

In the case where the angles α, β, γ is

$$\alpha = \beta = \gamma = \frac{\pi}{2}. \tag{1}$$

the joint is called common cardan joint or simpler just cardan joint, but in the case where

$$\alpha = \gamma = \frac{\pi}{2}, \beta \neq \frac{\pi}{2}. \tag{2}$$

the joint is with the crosses parts non-perpendicular.

Structurally speaking [1], [5], [7]. the kinematic pairs A', B' (see Fig. 3) are passive and then, structurally and cinematically speaking, they can be replaced with the element 2 from Fig. 7.

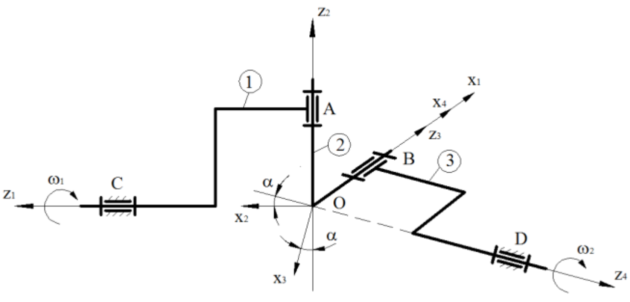

Fig. 7. 4R Asymmetrical spherical quadrilateral mechanism.

So are considered the bearings C and D and the concurrent rotation axes $Oz_i, i=1,2,3,4$.

The axes are being concurrent in the points $O_i, O_i', i=1,2,3$ and they coincide, so the mechanism from Fig. 7 becomes an RCCC mechanism where:

$$\boldsymbol{\sigma_i = 0, s_i = 0, i = 1,2,3,4.} \qquad (3)$$

If the angles α_i , $i=1,2,3$ are fulfilling the condition

$$\boldsymbol{\alpha_i = \frac{\pi}{2}.} \qquad (4)$$

then the cardan joint is called normal cardan joint.

For such a joint ($\theta_1^0 = 0$) one considers that $\alpha_4 = \pi - \alpha$ and from Fig. 2. results:

$$\boldsymbol{\theta_2^0 = \frac{\pi}{2}, \theta_3^0 = \frac{3\pi}{2} + \alpha, \theta_4^0 = \frac{\pi}{2}.} \qquad (5)$$

Consequently the study geometrical (technological) deviations of the cardan joint mechanism starts from the RCCC spatial quadrilateral mechanism, from which the customization.

IV. Identifing the Geometrical (Technological) Deviations

From the kinematic scheme of the RCCC mechanism from Fig. 2. results that, the component elements have one form namely that shown in Fig. 8. for $\sigma_i \neq 0$ respectively in Fig. 9. for $\sigma_i = 0$, case in which the points O_i, O_{i+1}' are overlaid.

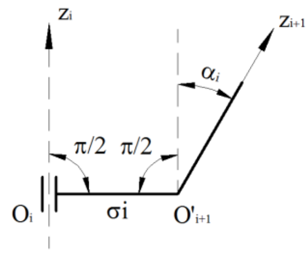

Fig. 8. The RCCC mechanism for $\sigma_i \neq 0$.

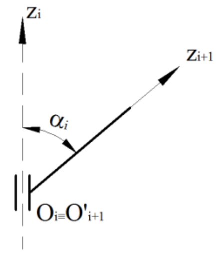

Fig. 9. The RCCC mechanism for $\sigma_i = 0$.

Other constructive variant of the RCCC mechanism is given in Fig. 10. case in which the elements 1 and 2 have the representations from Fig. 11. and Fig. 12.

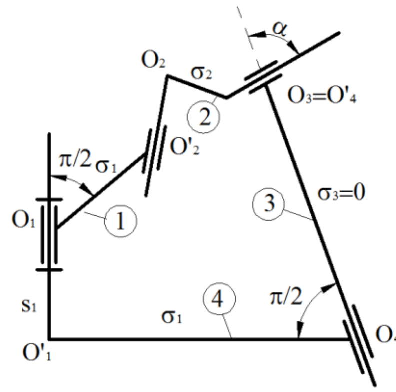

Fig. 10. Constructive variant of RCCC mechanism.

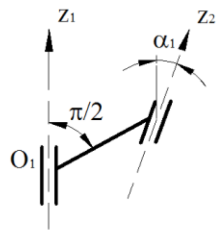

Fig. 11. Constructive variant for the element 1.

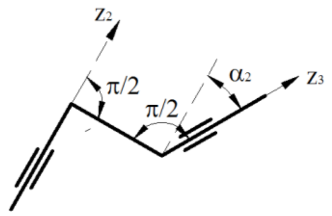

Fig. 12. Constructive variant for the element 2 .

In representations from Fig. 8., Fig. 9. and Fig. 10. where by O_i, O_{i+1}' was noted the poins where the perpendicular common of the axes $O_i' z_i, O_{i+1}' z_{i+1}$ these axes intersects.

In the points O_i is not necessary to be located and kinematic pair and in this sense kinematic scheme from

Fig. 10. may have the representation from Fig. 13. where the elements 1 and 2 are provided with forks.

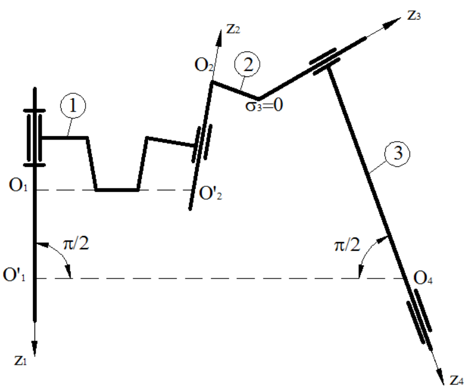

Fig. 13. Constructive variant of the RCCC mechanism.

A kinematic diagram that represents a mechanism with one cardan joint, with all geometrical deviations possible [2]., is presented in Fig. 14.

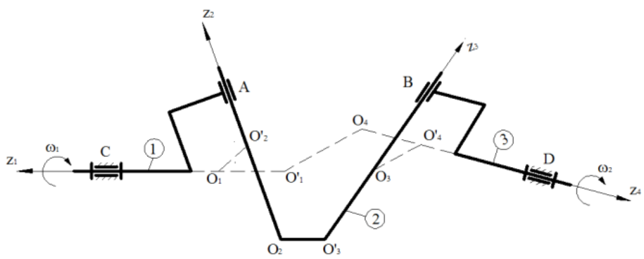

Fig. 14. Geometrical deviations.

These deviations are small and fulfill the condition:

$$\begin{aligned} \alpha_i &= \frac{\pi}{2} + \Delta\alpha_i, i = 1,2,3, \alpha_4 = \pi - \alpha \\ \sigma_i &= O_i O'_{i+1}, i = 1,2, \sigma_4 = O_4 O'_i \end{aligned} \quad (6)$$

1. *The angularly deviation of the main shaft bracket is defined by the parameter* $\Delta\alpha_1$ *and the smoothness deviations for the same bracket is given by the parameter* σ_1.

2. *The angularly deviation of the cardanic cross 2 is given by the parameter* $\Delta\alpha_2$ *and also the deviation from smoothness is given by the parameter* σ_2.

3. *The angularly deviation of the driven shaft bracket 3 is given by the parameter* $\Delta\alpha_3$ *and the smoothness deviation is given by the parameter* σ_3.

4. *The angularly deviation of the driven shaft 3 depending on the driving shaft 1 is given by the parameter* σ_4.

V. CONCLUSIONS

As shown in default of shafts 1 and 3 are known the points (see Fig. 14.) $O_4, O'_1, O_1, O'_2, O_2, O'_3, O_3, O'_4$ are overlaid with point O (see Fig. 9.) and the cylindrical kinematic cpairs A , B and D (see Fig. 14.) become rotation kinematic pairs (there are no displacements s_2, s_3, s_4 along the axes Oz_2, Oz_3, Oz_4 .

The existence of technical deviations conducts to the displacements $s_i, i = 1,2,3,4$ and by blocking those the excess efforts from the rotation kinematic pairs A, B , C , D appear (see Fig. 7.).

In order to determine these displacements it is first necessary to calculate the angularly parameters $\theta_2, \theta_3, \theta_4$ variation depending on the angle θ_1 .

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