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OPTIMAL DESIGN OF TRUSS GIRDER

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Abstract**—**In the literature the optimal design compose in general one aim function what are the mass, volume or costs of material in case of truss structure. The costs of joints are difficult to estimate. In this work we show the optimization with multiobjective functions. In this case the objective functions are the mass of truss girder and the deflection of joints. The design constraints are prescribed by EUROCODE 3**.**

Keywords**—**Optimal design, multiobjective function, design constraints, cost function.

I. Introduction

WE can find lots of examples for the optimization of girder truss in a technical literature. The problems of optimizations can put into two groups:

1) the geometrical shape of structure is given and have to calculate the cross section of the beams,

2) the geometrical shape of structure is not given, so we have to optimize the shape of structure (topology).

The [1] and [9] literatures write about the method and results the optimization of the cross-section of truss girders in a plane. The authors chose the mass of structure what was usual in that time. The design constrictions were stress, displacement, eigenfrequency and mass of structure. The novelty results show Martinez and his mates on the field of optimization of shape and measurement of truss girder [2].

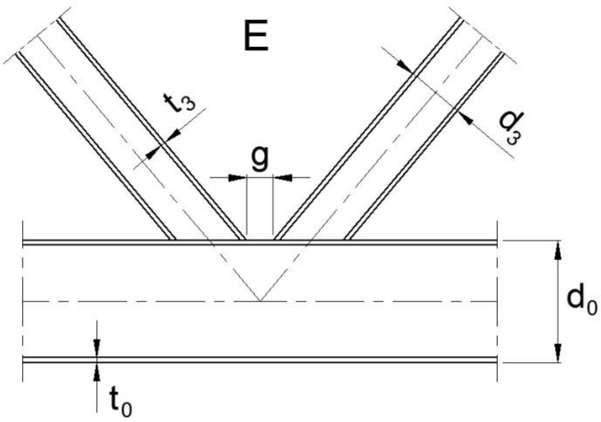

Fig. 1. One form of joint

The joint forming cost can be read in the book [7] that shows the Fig.1.

By the solution of the shape optimization are using become the algorithms of evolutionary optimization [3]. About the topology optimization wide-ranging survey is given in the book of Bendsoe [4].

II. Several Effective Methods of Optimization

A. The optimization

The general mathematical description of an optimization problem is defined as to minimize an objective function *f(x)*

$$\begin{aligned} &\min f(x), \rightarrow x \in E^n \\ &0 \le g_j(x), \rightarrow j = 1,2,...,m \\ &0 = h_j(x), \rightarrow j = m+1,...,p, \end{aligned} \qquad (1)$$

subject to *m* linear and/or nonlinear inequality constraints, *p-m* linear and/or nonlinear equality constraints and $\boldsymbol{x}=[x_1, x_2,..., x_n]^T$ the variables (design parameters) [5]. Types of optimizations are often as follows:

1) Optimization of the geometry In the case of dimension (geometry) optimization often the dimensions of cross sections are computed.

2) Shape optimization

Traditionally, in shape design of mechanical bodies, a shape is defined by the oriented boundary curves of the body and a shape optimization the optimal form of these boundary curves is composed.

3) Optimization of the topology.

4) Optimization of the material structure.

B. Multiobjective optimization

Multiobjective optimization is a vector optimization, each element of which represents one of the objective functions being optimized. Effective methods for multiobjective optimization are listed here [6]:

1) Hierarchical optimization method

The objective functions are arranged in order of importance which is based on the preference of decision maker. The objective functions in level 1 are considered as the most important and optimized first.

$$\mathbf{Min} \quad \mathbf{f_1}\left(\bar{\mathbf{x}}\right) \qquad (2)$$

In the level 1 the optimum design variables

$$\bar{\mathbf{x}}^{(1)} = \left\{ \mathbf{x}_1^{(1)}, \mathbf{x}_2^{(1)}, ..., \mathbf{x}_n^{(1)} \right\} \qquad (3)$$

and the optimum objective function is $f_1(\bar{x}^{(1)})$. For the following levels

$$\mathbf{Min} \quad f_i\left(\bar{x}\right) \quad i = 2,...,m \tag{4}$$

the additional constraints are needed

$$f_{j-1}\left(\bar{x}\right) \le \left(1 + \frac{\varepsilon_{j-1}}{100}\right) f_{j-1}\left(\bar{x}^{j-1}\right) \quad j = 2,...,i \tag{5}$$

where ε_{j-1} is a relaxation coefficient. If there are m levels the final optimum design variables are the next

$$\bar{x}^{(m)} = \left\{x_1^{(m)}, x_2^{(m)}, ...x_n^{(m)}\right\} \tag{6}$$

2) Goal programming

Goal programming is very effective method for multiobjective optimization. The decision maker has to set an inspiration level for each of the objective functions, i. e. giving a target value. On the basis of decision maker's preference the objective functions may be rank – ordered into priority levels, P1, P2,… P1 is preferred to P2, described as P1>>P2. In the same manner P2>>P3, P3>>P4,…. The new deviation variables di+ and di- are defined to measure the overachievement and underachievement of the i-th goal. In each priority level the summation of the deviation variables multiplied by its weighting factor is considered as an achievement function. The optimum solution may be obtained by minimizing the achievement function in order of priority levels. It is desirable to obtain a design whose performance matches the aspirations as close as possible.

3) Weighting method

The weighting factors represent the relative importance of the objective functions from the decision maker's view point. Because there is no practical analytical method to define the weighting factors now they are selected by experience. The weighting method has been applied extensively in engineering design because of its simplicity. We use this method too.

III. DESIGNING OF TRUSS WITH EUROCODE 3

The employments of the beams of truss girder are tension and compression. The maximal normal stresses in the bars have to be smaller or equal as the allowed value. At the compressed beams of girder truss are existed the danger of bending. So the forces of the beams have to be smaller than the counted value of standard (EUROCODE 3). In case of the centralized compression is the bending in a plane the competent, so have to check on bending resist of compressed beams

$$\frac{N_{ed}}{N_{b,Rd}} \le 1, \tag{7}$$

where N_{ed} is the compressive force, $N_{b,Rd}$ is the design value of compression force. The bending resist of the centralized compression of beam generally have to determine in two perpendicular planes.

$$N_{b,Rd} = \frac{\chi \beta_A A f_y}{\gamma_{M1}}, \tag{8}$$

where χ is reduction factor of bending mode, $\beta_A=1$ if the cross-section is 1, 2 or 3 class, $\beta_A=A_{eff}/A$ if the cross-section is 4 class, where A is the area of cross-section and A_{eff} is the effective area of compression cross-section (EC3/1.5), f_y is a yield stress limit of material, γ_{M1} is the safety factor. In case of constant cross-section and constant compressed force χ depend on $\bar{\lambda}$ what is the reduced slenderness of beam (9)

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, \text{ but } \chi \le 1, \quad \text{where}$$

$$\Phi = \frac{1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2}{2}, \tag{9}$$

where α is a shape factor that value give the norm. The reduced slenderness ratio we could calculate with the next function (10):

$$\bar{\lambda} = \sqrt{\frac{\beta_A A f_y}{N_{cr}}} = \frac{\lambda}{\lambda_1}\sqrt{\beta_A}, \tag{10}$$

where $\lambda=l_{cr}/i$ is a bending plane counted slenderness ratio (this value have to count in two plane in general), l_{cr} the bending length of beam, i is a radius of gyration of cross-section head axis of perpendicular bending plane and N_{cr} is the break force. The Fig. 2. shows the calculated values of the reduction factor [8].

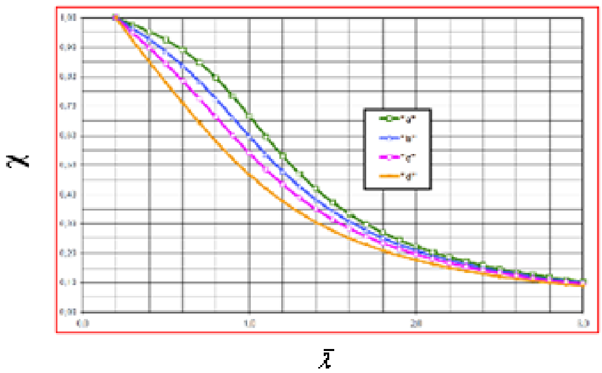

Fig. 2. The calculated values ($\bar{\lambda}$) of reduced slenderness ratio

The λ_1 value we calculate with the next function (11):

$$\lambda_1 = \pi\sqrt{\frac{E}{f_y}} = 93,9\sqrt{\frac{235}{f_y}} = 93.9\varepsilon. \tag{11}$$

At the optimization of truss girder in general the objective function is the mass of structure, the costs of material or the volume of structure, because the cost of making of joint are very difficult to define and reference data are not given. The design constraints concern to the stresses, stability and geometrical sizes. The unknowns are the cross-sections of beams what values could be discrete or continuous. In the next we would like to show the optimal design of truss girder with several objective functions. In this case the objective functions will be the mass of structures and the displacements of joints. The design constraints are the displacements of joints, the stresses in the beams of structure and the stability of the compressed beams. To compose form mathematically the optimization task we have to define the minima of $F(x)$ function in case of the inequality and equality design constrains

$$\mathbf{F(\underline{x}) = \sum_{1}^{n} w_i f_i(\underline{x}), \qquad g_j(\underline{x}) \le 0, \qquad j = 1,...,m,}$$

$$\mathbf{h_j(\underline{x}) = 0, \qquad j = m+1,...,p,} \tag{12}$$

where w_i are weighting factors, $f_i(x)$ is the i-th objective function of multiobjective functions, $g_j(\underline{x})$ and $h_j(\underline{x})$ are the inequality and equality design constrains. To solve this problem there are several methods, but in technical fields the simple weighting factors are used. The point is that each objective function has to be multiplied by different weighting factor that has importance on the given objective function. So we can create from the multiobjective function problems one objective function task with holding the design constrains to take care that (13)

$$\mathbf{\sum_{1}^{n} w_i = 1, \qquad w_i \ge 0.} \tag{13}$$

For the solution of multiobjective functions optimization there are different mathematical methods.

IV. The Optimization of Type “A” Girder

We show the optimizations of two different types truss girders.

A. Compose of the objective functions

As in Fig. 3. can be seen the truss girder has eleven bars. The objective functions are the mass of the structure and the displacements of joint “C”. At the optimization of displacements are important rule, because its value have to known, or have to restrict. The structure of truss girder m mass we can calculate with the next connection:

$$\mathbf{m = f_1(x) = \sum_{i=1}^{11} A_i l_i \rho} \tag{14}$$

where A_i is each beam cross-section, l_i is the length of beams and ρ is the density of the material. The displacement of joint we calculate with Betti theory, so the displacement of “C” joint is

$$\mathbf{w_c = f_2(x) = \sum_{i=1}^{11} l_i \frac{N_i n_{Ci}}{A_i E}} \tag{15}$$

where N_i is the force acting in the beam of the F_1 and F_2 forces, n_{Ci} is the force in the beam of the acting of a $Q_C=1$ force and E is a Young modulus of the material. There are two objective functions, so we can solve it with the weighting factor method. The weighting factors have to take consideration, how important we think the problem from the point of view.

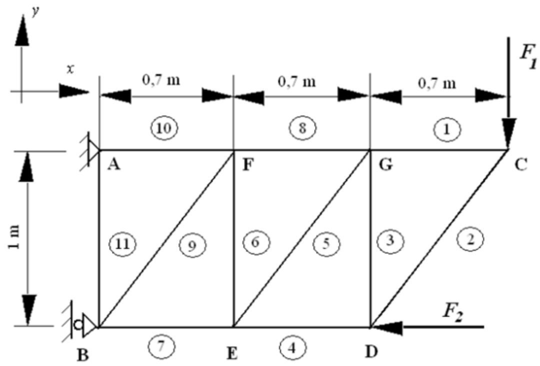

Fig. 3. The structure of truss girder “A”, the forces are vertical and horizontal

B. Compose of the design constraints

For the displacement of joint “C” we prescribe that it should be smaller than the given displacement

$$\mathbf{w_{max} \le w_{pr}} \tag{16}$$

where w_{pr} is the prescribed displacement of joint. The employment of beams of truss girder is tension or compression. So the normal stress of each beam must to be smaller than the given stress

$$\mathbf{\sigma_{max} = \frac{N_i}{A_i} \le \sigma_{pr}} \tag{17}$$

where σ_{pr} is the prescribed stress, N_i is the acting force.

In the compressed beams of truss girder is the bending danger, so the acting force in the beam must to be smaller than the prescribed one of the norm (EUROCODE 3) as here

$$\mathbf{N_i \le N_{b,Rd},} \tag{18}$$

where $N_{b,Rd}$ is the design value of compression force.

In the earlier composed nonlinear task of optimization we show with the following truss girders.

C. Results of truss girder type “A”

The truss girder is shown in the Fig. 3. is statically determined. Data are the next: E=210 (GPa), f_y=235 (MPa), F_1=4 (kN), F_2=5 (kN), β_A=1; γ_{M1}=1,1, ρ=7850 (kg/m^3), σ_{pr}=200 (MPa), w_{pr}=40 (mm), b= 8 (mm), x_{imin}= 40 (mm), x_{imax}= 200 (mm). In the Table I are the height of beams, where x_1 is the height of beams 1, 8 and 10-th., x_2 is the height of beams 4 and 7, x_3 is the height of beams 2, 5 and 9, and x_4 is the height of beams 3, 6 and 11-th (A_i=b*x_i), where “b” is the width of beams.

We have studied that how the changing of weighting factor (the importance of objective function) do change the optima. From the results it appears that multiobjective functions we can design such structure that is the best suit requirements.

It can be seen in TABLE I. that the restricting of prescribe of deflection is led to the increase of mass of structure. With the changing of the weighting factor we can give satisfaction to the different requirements.

TABLE I
DIMENSIONS OF TYPE "A" TRUSS GIRDER

weighting factor (w_1)	1	0.75	0.25	0
weighting factor (w_2)	0	0.25	0.75	1
mass (kg)	36.9	37.6	40.0	103.0
displacement (mm)	38.5	35.7	31.6	31.1
x_{1min} (mm)	43.7	40.0	42.9	141.8
x_{2min} (mm)	163	176.8	200	200.0
x_{3min} (mm)	40.0	40.0	40.1	170.0
x_{4min} (mm)	40.0	40.4	40.0	138.7

V. RESULTS OF TRUSS GIRDER TYPE "B"

In Fig. 4. can be seen truss girder that are F_1 and F_2 forces loaded and it has eleven bars. We show its optimal design. Now the bars are square cross-sections, so the unknowns are the cross-sections of bars. The geometrical form is given.

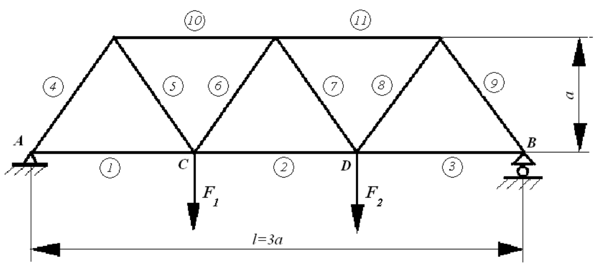

Fig. 4. The structure of truss girder "B"

A. Compose of the objective functions

In this case the objective functions are the mass of the structure and the displacements of joint "C". The structure of truss girder *m* mass we can calculate with (14) and deflection of joint "C" with (15).

B. Compose of the design constrains

To the displacement of joint "C" we prescribe that should be smaller than the given value.

$$\mathbf{w}_{max} \leq \mathbf{w}_{pr} = \mathbf{C}_w \mathbf{l} \qquad (19)$$

where C_w is a bending parameter and l is the long of beam. The employment of beams of truss girder is tension or compression. So the normal stresses have to be smaller in each beam than given by EUROCODE 3 (17). In the compressed beams of truss girder is the bending danger, so the force in a beam have to be smaller than the prescribed of norm (EUROCODE 3) (18). It is a nonlinear optimization.

C. Results of truss girder type "B" Fig. 4.

In Fig. 4. visible statically determined truss girder data are the following: a=1,5 (m); C_w=1/300; E=210 (GPa); f_y=235 (MPa); F_1=60 (kN); F_2=36 (kN); l=4,5 (m); Q_C=1 ; β_A=1; γ_{M1}=1,1; ρ=7850 (kg/m^3); σ_{pr}=200 (MPa). Value of χ is counted by computer according to EUROCODE 3.

We solved the task with the multiobjective optimal design method. Fig. 5. shows the minimal value of the optimal beams cross-sections.

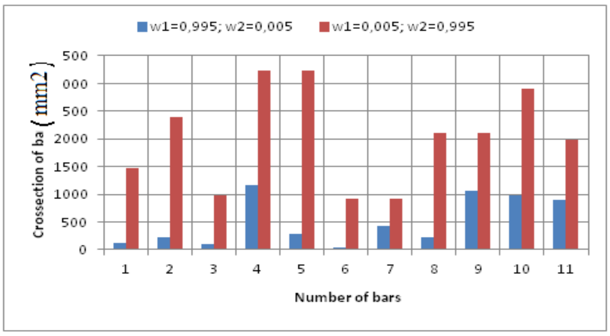

Fig. 5. The cross-section of beams of truss girder

On the left side column the mass of truss girder was the objective function in the majority of cases (w_1= 0.995) and the deflection was negligible (w_2= 0.005). We got the right side higher columns that the aim was the minima of deflection (w_2= 0.995). This aim could be achieved with a more stiffness and heavier structure.

VI. CONCLUSION

From the results appear that in case of several objective functions with the changing of weighting factor we can design such structures that is the best suit requirements.

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