ANNALS OF THE ORADEA UNIVERSITY
Fascicle of Management and Technological Engineering
ISSUE #1, MAY 2015, http://www.imtuoradea.ro/auo.fmte/

EQUIVALENCE BETWEEN BALL-ROTATING DISC IMPACT AND BALL-INCLINED PLANE IMPACT, PART I: EQUIVALENCE CONSTRAINTS, PLASTIC IMPRINTS COMPARISON

Stelian ALACI[1], Florina Carmen CIORNEI[2]
[1] Stefan cel Mare University, Suceava, e-mail:alaci@fim.usv.ro
[2] Stefan cel Mare University, Suceava, e-mai: florina@fim.usv.ro

Abstract**—**The paper aims to answer to the question whether the spatial impact between a free falling ball and the frontal surface of a rotating disc around a vertical axis, on one side, and the impact between the same ball and an inclined fix plane can be considered equivalent. An affirmative response to this problem would allow assimilating the disc-ball impact to a plane impact and the possibility of applying a simpler study method. The first part of the work presents the equivalence conditions and the comparison between plastic imprints of the ball obtained in the two cases are presented.

Keywords—impact, Coulomb friction, plastic indentation

I. Introduction

THE impact phenomenon is one of the most spectacular mechanical phenomena due to the noticeable effects caused by the appreciable energy released during time period of milliseconds or even less. The behaviour of a dynamical system where impact phenomenon occurs can be more or less complex depending on the hypothesis adopted in modelling the system. The literature contains numerous references concerning impact phenomenon, from elementary dynamics manuals to recent papers from journals of specialty, [1]-[35].

Two main directions were outlined in modelling the impact phenomenon: the first, considers an instantaneous collision between rigid bodies and the second, considers collision between deformable bodies. Acceptance of friction forces during impact gives a more complex character to study of impact phenomenon. For the case of plane impact with dry friction between two rigid bodies, Routh [24], proposed a graphical method that permits a relatively simple study of the phenomenon.

The present work tries to verify if the collision between a rotating disc and a metallic ball can be approximated by a plane impact. The relative motion between the two bodies is obviously a spatial one if it is considered that after impact, due to the friction between the two bodies the ball will have a rotation around and axis and it is less probable that this axis will be parallel to the disc's rotation axis. To obtain the answer to this issue, an existing device was updated with new elements, the main part being a metallic disc that can rotate around a vertical axis. The disc is driven via a belt transmission by a d.c. motor. The operation principle of the test rig consists in launching in free fall a ball from a known height; it collides the horizontal surface of a rotating disc while a non-contact tachometer gauges the rotating speed of the disc.

Two test-rigs were designed, in order to response to the question if the spatial impact between a ball and a disc can be assimilated as a plane impact between a ball and an inclined plane, to obtain the same relative impact motion. By comparing the postimpact relative motions and plastic imprints for the two cases, respectively, one can sketch the conclusions concerning the possibility of equivalence between the two motions.

II. Equivalence Conditions between the Ball-Rotating Disc Impact and Ball-Inclined Plane Impact

One of the most effective methods for dynamical analysis of mechanical systems – and both mechanisms and robots are included here, is the dynamical analysis of multibody systems, [25], [36]. A distinct multibody dynamic problem consists in systems with impact phenomena occurrence. The impact phenomenon is characterised by sudden variation of kinematics parameters and therefore the incidence of significant stresses in the joints of the system, [22], [37].

The consequence of the great values of impact forces is, in most of the cases, the occurrence of remnant plastic imprint of contact regions. In the case when the variation of kinematical parameters is rapid, the intensity of the stresses can be that high that leads to irreversible damage of contacting surfaces. A comprehensive study upon impact behaviours of a dynamical system with complex geometry is extremely complex due to the multitude of parameters to be considered. From this reason, most of the impact studies consider simple geometrical systems, requiring a reduced number of parameters in describing the dynamical behaviour, [19]-[21].

Taking into account that the impact is essentially a mechanical contact accomplished during a very short time period, than the effect of the contact will be strictly local. From this reason, in the study of impact phenomena occurring in complex systems, only the local geometry of the potential impact point vicinities is established. Thus, it is adequate to study the impact behaviour of two simple bodies with curvature radii identical to the ones of the real contacting surfaces. Finding the effects of impact for the simple system, these results can be applied for the boundary surfaces of actual parts. According to Saint-Venant’s principle, at sufficiently large distances from the loads, the manner the forces are applied doesn’t matter, and thus the principle of described method is justified. These considerations explain the numerous papers dedicated to impact between bodies limited by plane, spherical or cylindrical surfaces, between which more or less complex motions happen. The percussions study can be made through two main methodical categories, depending on the adopted hypothesis.

The first technique (*stereomechanic*) accepts that the impact phenomenon is instantaneous and takes places between perfectly rigid bodies. The procedure allows finding the post-impact kinematical and dynamical states of the system as functions of initial states. The variations are obtained by writing the equations characteristic to dynamical theorems, theorems expressing finite momentum variations, moment of momentum variations and kinetic energy corresponding to ending and starting of impact process, respectively.

The variations of different kinematical parameters are described by the ratio between the values of the parameter at the end and at the beginning of impact, respectively. Among these parameters, the most important is the coefficient of restitution, *e*. The kinematical definition of this coefficient is made based on Fig. 1:

$$\mathbf{e} = -\frac{\left(\mathbf{v}''_2 - \mathbf{v}''_1\right)\cdot\mathbf{n}}{\left(\mathbf{v}'_2 - \mathbf{v}'_1\right)\cdot\mathbf{n}} \qquad (1)$$

The second technique of approaching an impact process is obtained adopting the hypothesis of finite time period of impact course, [10],[19],[29]. During the whole impact process, all parameters, both kinematical and dynamical ones, present continuous variation.

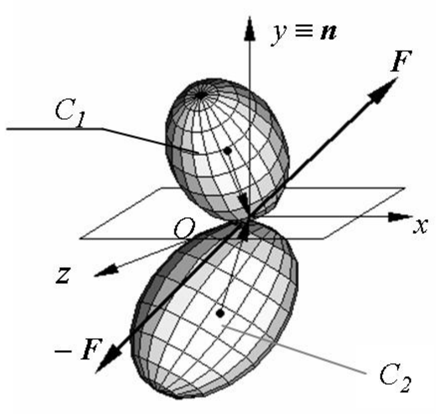


Fig. 1. Oblique collision of two bodies

The significant advantage of this last method consists in the possibility of estimation of forces developed between the two bodies implied in impact. In this case, the assumption of rigid bodies is abandoned and it is accepted that they are deformable. From Fig. 2, observing the impact process, one considers that it starts when the first points of the two bodies are contacting, then the impact continues and the approach between the two bodies increases till the moment of reaching the maximum value of the normal approach, $y=y_{max}$. At this moment, the relative velocity between the two bodies is zero and this moment, t_c corresponds to maximum approach. After this instant, the restitution phase starts when the approach between the two bodies decreases. The restitution process ends at the moment, t_f.

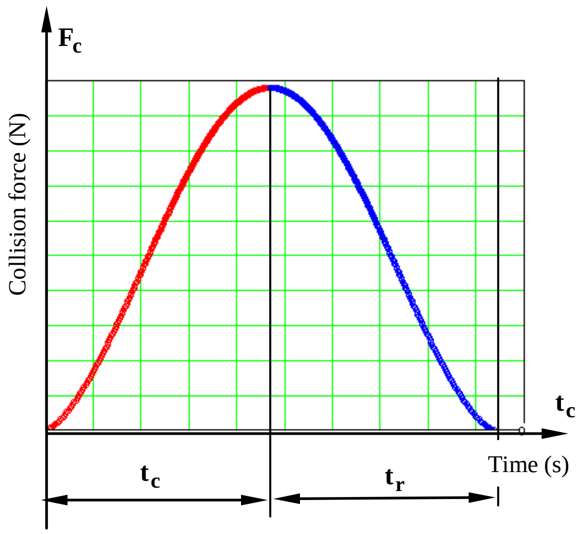


Fig. 2. Phases of collision process

Friction is not a process to be ignored in dynamical analysis of system, except for an initial stage. Dry friction is characterized by the lack of lubricant between the colliding bodies and thus a direct contact occurs between bodies.

Friction presence makes more difficult the study of impact phenomenon. A suggestive example was given by Kane, [38]. He analyses the impact between a double pendulum and a rough horizontal plane.

After a convenient selection for the initial pendulum position and for the values of coefficients of friction from joints and of coefficient of restitution, Kane proves that the final kinetic energy of the pendulum is greater than the initial one.

The cause of this paradoxical is the manner the coefficient of restitution is defined. To surpass this inadequacy the coefficient of restitution must be defined according to Poisson, using a dynamic parameter, namely percussion, defined as the time integral of impact force. Thus, the coefficient of restitution is describes as the ration between the percussion of restitution phase and the percussion of compression phase.

$$e_P = \frac{\int_{t_c}^{t_f} \mathbf{F}\cdot\mathbf{n}}{\int_0^{t_c} \mathbf{F}\cdot\mathbf{n}} = \frac{P_n^f - P_n^c}{P_n^c} \quad (2)$$

The notion of percussion proved a valuable instrument in the study of impact with friction. In 1890, Routh, [24], proposed a method extremely effective for the study of plane impact with friction, namely the method of percussions plane.

Since the impact between the free falling ball and the rotating disc is a spatial collision, it cannot be studied by applying the Routh method.

In the case that this collision possibly will be equivalent to the plane impact between a free falling ball and an inclined plane, the initial ball-disc impact could be analyzed using the Routh method for the equivalent impact ball-inclined plane.

III. KINEMATICAL EQUIVALENCE OF COLLISIONS

The present paper aims to assimilate the ball-disc impact to a ball-inclined plane and to this end the following condition must be confirmed:

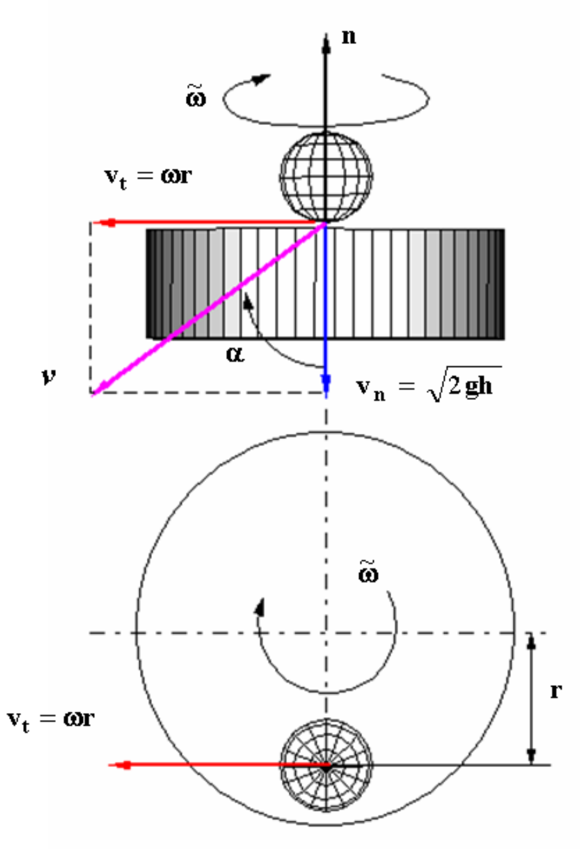
1) *Plastic imprints coincidence* between the marks resulted from the two collisions;

2) *Kinematical coincidence:* in both cases, the impact relative velocity between the ball and the impact surface must have the same value and the same position with respect to the normal to the surface.

A similar form of previous condition requires that the tangential velocities and the normal velocities should be equal, respectively, expressed mathematically as:

$$\begin{aligned} V_t &= v_t \\ V_n &= v_n \end{aligned} \quad (3)$$

where V_t, V_n and v_t, v_n are the tangential and normal components for the collision with the inclined plane and the rotating cylinder, respectively. Denoting by ω the angular velocity of the cylinder, by r the radius of the point from the surface of the cylinder and by α the tilt angle of the inclined plane, (3) become:



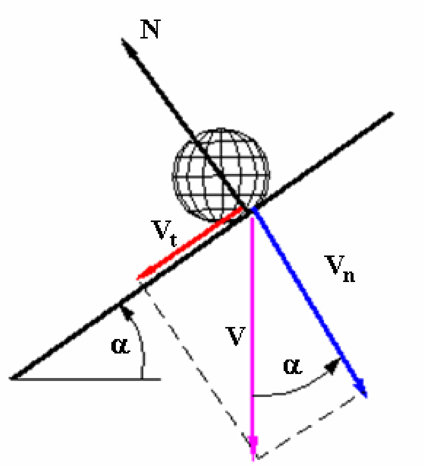


Fig. 3. Kinematical equivalence of the two collisions

$$\begin{cases} V\sin\alpha = \omega r \\ V\cos\alpha = v \end{cases} \quad (4)$$

From (4) the angle of the inclined plane and the velocity the ball must have when collides it can be found:

$$\begin{cases} V = \sqrt{v^2 + \omega^2 r^2} \\ \alpha = \mathrm{atan}(\omega r / v) \end{cases} \quad (5)$$

For the free falling ball, the height of launching it can be obtained, $V = \sqrt{2gH}$.

The fact that the ball is let free for $\mathbf{H_{max} < 1.5m}$, leads to a limit value for the disc angular velocity in the ball-disc impact. From the relation:

$$\mathbf{2gH_{max} = 2gh + \omega^2 r_{max}^2}\,, \qquad (6)$$

the maximum angular velocity the disc might have can be found.

IV. Comparative Analysis of Post-Impact Indentations

The post-impact plastic indentations corresponding to the collision of the ball with the rotating disc and to the collision of the ball with the inclined plane ware scanned using the laser profile-meter NANOFOCUS. In Fig. 4 there are presented the photography of an indentation, the 3D image obtained using the scanner and the tangential and radial profiles.

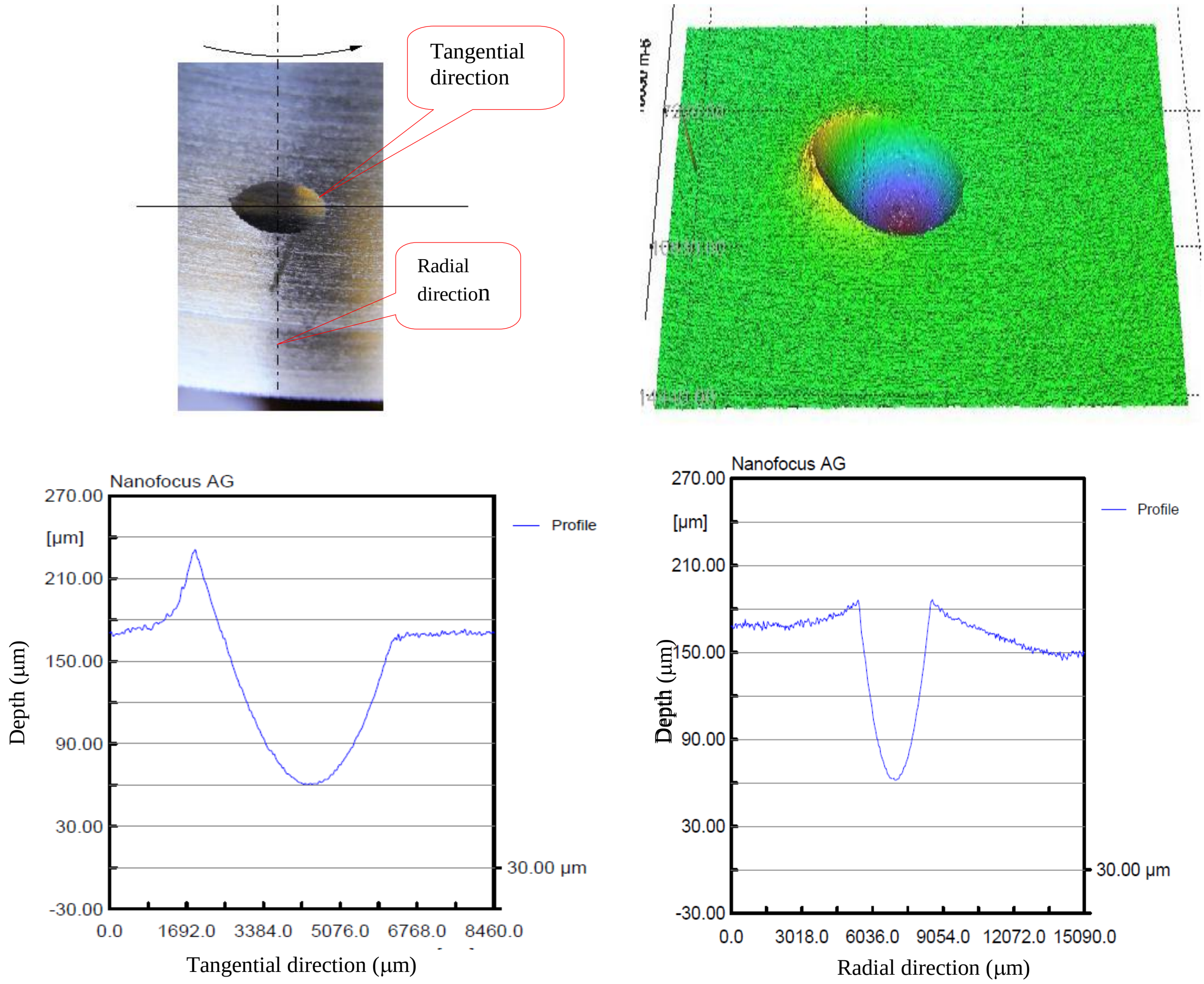


Fig. 4. Photography, 3D image, tangential and radial profiles for an indentation on an aluminum disc

In order to compare two scanned profiles, [39], from the entire scanned profile only a zone was separated, plastically deformed, that afterwards was approximated by a parabolic function, [40], Fig. 5.

Using (5), for two angles of the inclined plane, the rotation velocity of the disc was found. In a first stage, the repeatability of the results corresponding to two identical collisions ball-disc, Fig. 6, and Fig. 7, was verified. The very good agreement between the graphs can be observed.

In Fig. 8 and Fig. 9 there are presented the tangential profiles of the plastic indentations from the disc (red) and from the surface of the inclined plane (blue). In this plots the concordance between the axial profiles of the two imprints is very good, except for the zone in the vicinity of the free surface where the pile-up dimensions differ.

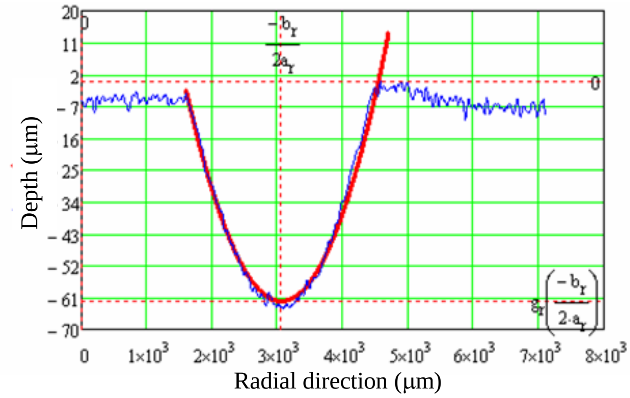


Fig. 5. Approximation by a parabolic plot of selected points

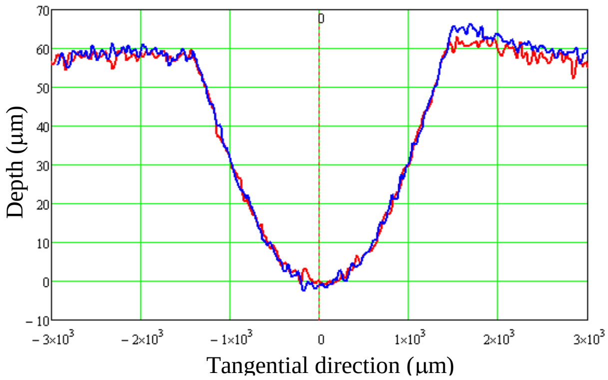


Fig. 6. Comparison between tangential profiles for $\mathbf{n} = 480\,\mathbf{rot/min}$

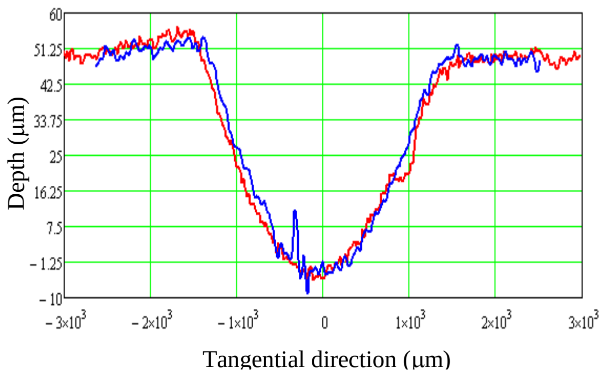


Fig. 7. Comparison between tangential profiles for $\mathbf{n} = 680\,\mathbf{rot/min}$

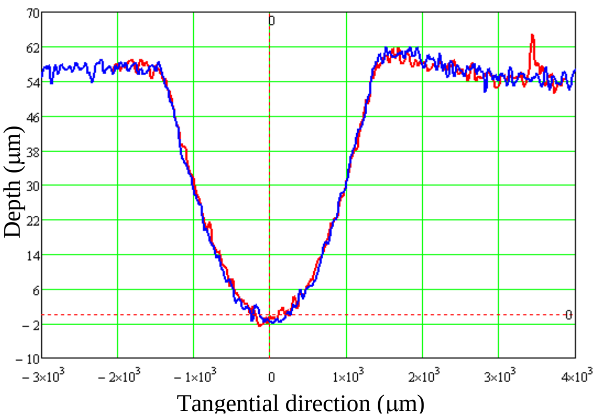


Fig. 8. Comparison between tangential profiles for inclined plane and rotating disc for $\mathbf{n} = 480\,\mathbf{rot/min}$

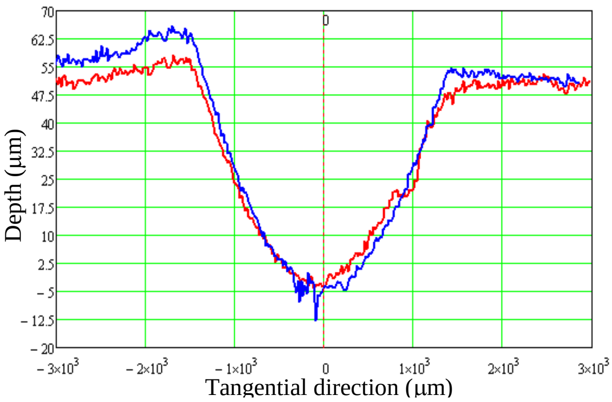


Fig. 9. Comparison between tangential profiles for inclined plane and rotating disc for $\mathbf{n} = 680\,\mathbf{rot/min}$

V. CONCLUSION

The present work analyses the possibility of similarity between a spatial collision between a free falling ball and a rotating disc and a plane collision between the same ball and an inclined plane. An affirmative answer to this subject would substantially simplify the study of spatial impacts.

From qualitatively point of view, the paper shows that after impacting the rotating disc, the ball performs a rotation around an horizontal axis and the condition that after impact, the ball's motion should be a plane one, is fulfilled.

From quantitative perspective, the comparison assumes two aspects. First, there were identified the conditions required for considering identical relative motions between the colliding bodies. More explicit, the velocity and the incidence angle for the two impacts must be the same. To satisfy these conditions, two test rigs were designed and constructed. For both devices, the height of free fall of the ball can be adjusted and for the first test-rig, the disc angular velocity can be modified and experimentally evaluated while for the second test-rig, the tilt angle of the plane can be precisely regulated.

For both collisions the duration of flight of the ball from impact instant till the second impact on an aluminium plate is measured. Another experimental parameter is the distance between the marks of the collision ball-disc or ball-plane on one side and ball-aluminium plate, on the other side. With these parameters found experimentally and using own programs, the velocity and reflection post-impact angle can be found.

A second aspect aimed in experimental researches consists in analyzing geometric characteristics of post impact plastic indentations for the two collision cases. This study offers information upon the work of plastic deformation needed during the impact process.

From the experiments completed the following conclusions can be drawn:

1) After the impact, the velocity of the ball centre is smaller $\cong 25\%$ *in the case of ball-disc collision than the inclined plane case;*

2) The reflection angle is smaller in the case of ball-disc impact;

3) The tangential profiles are practical identical for the two values of angular velocities of the disc. More, the indentations corresponding to two impacts at the same angular velocity of the disc, are very close, showing a good results repeatability

From the above considerations, from the point of view of plastic imprints, the ball-rotating disc impact can be equivalent to a collision between the same ball and an inclined plane. Though, considering that the rotation energy the ball receives after impact is much smaller comparatively to the translation kinetic energy and the work of damping or plastic deformation, for preliminary calculus the ball-disc impact can be solved, with a good approximation, using the ball-inclined plane model.

ACKNOWLEDGMENT

The authors acknowledge financial support from the project "Integrated Center for research, development and innovation in Advanced Materials, Nanotechnologies, and Distributed Systems for fabrication and control", Contract No. 671/09.04.2015, Sectoral Operational Program for Increase of the Economic Competitiveness co-funded from the European Regional Development Fund.

REFERENCES

[1] S. Djerassi, "Collision with friction; Part A: Newton's hypothesis". *Multibody System. Dynamics* 21, pp. 37–54, 2009.
[2] S. Djerassi, "Collision with friction; Part B: Poisson's and stronge's hypotheses". *Multibody Syst. Dyn*. 21, 2009, pp. 55–70.
[3] S. Dubowsky, F. Freudenstein, "Dynamic analysis of mechanical systems with clearances. Part 1: Formulation of dynamic model", *Journal of Engineering for Industry*, Series B 93(1), pp. 305-309, 1971.
[4] S. Dubowsky, F. Freudenstein, "Dynamic analysis of mechanical systems with clearances. Part II: Dynamic response", *Journal of Engineering for Industry,* Series B 93 (1), pp. 310-314, 1971.
[5] S. Faik, H. Witteman H., "Modelling of Impact Dynamics: A Literature Survey", *North American ADAMS User Conference*, June 2000.
[6] P. Flores, J. Ambrósio, J.C.P. Claro, H. M. Lankarani, "Kinematics and Dynamics of Multibody Systems with Imperfect Joints: Models and Case Studies". *Lecture Notes in Applied and Computational Mechanics*, vol. 34. Springer, Berlin, 2008.
[7] P. Flores, "Modeling And Simulation Of Wear In Revolute Clearance Joints", *Multibody Systems. Mech. Mach. Theory* 44(6), pp. 1211–1222, 2009.
[8] P. Flores, J. Ambrósio, "On the contact detection for contact-impact analysis in multibody systems" *Multibody Syst. Dyn.* 24(1), pp 103–122, 2010.
[9] P. Flores, R. Leine, C. Glocker, "Modeling and Analysis of Planar Rigid Multibody Systems with Translational Clearance Joints Based on the Non-Smooth Dynamics Approach". *Multibody Syst. Dyn.* 23, pp 165–190, 2010.
[10] P. Flores, M. Machado, M.T. Silva, J.M. Martins, "On the continuous contact force models for soft materials in multibody dynamics", *Multibody System Dynamics*, vol. 25, pp. 357-375, 2011.
[11] W. Goldsmith, *Impact, The Theory and Physical Behaviour of Colliding Solids,* Dover Publication. Applications, Springer 2001.
[12] R. C. Hibbler, *Engineering Mechanics: Dynamics*, Prentice Hall, 2012.
[13] L. A. Huang, *Concise Introduction to Mechanics of Rigid Bodies: Multidisciplinary Engineering* (Springer), 2011.
[14] K. H. Hunt, F.R. Crossley, "Coefficient of Restitution Interpreted as Damping in Vibroimpact", *Journal of Applied Mechanics*, 7, 440-445, 1975.
[15] Y. Hurmuzlu, D.B. Marghitu DB, "Rigid body collision of planar kinematic chain with multiple contact points", *The International Journal of Robotics Research* 13, pp. 82–89, 1994.
[16] C. Iacob, "Asupra generalizării teoremei lui Lazare Carnot în cazul ciocnirii a două rigide libere", *Studii şi cercetări de mecanică Aplicată,* 43, Bucharest, , pp. 5-6, 2004.
[17] K. L. Johnson, *Contact Mechanics*, Cambridge University Press, 1985.
[18] T. R. Kane, D.A. Levinson, *Dynamics: Theory and Applications*, McGraw-Hill, New York, 1985.
[19] H. M. Lankarani, P.E. Nikravesh, "A contact force model with hysteresis damping for impact analysis of multibody systems", *J. Mech. Des.* 112, pp. 369-376, 1990.
[20] H. M. Lankarani, P. E. Nikravesh, "Canonical Impulse-Momentum Equations for Impact Analysis of Multibody Sistems". *Trans ASME, Jnl. Mech. Design*, Vol.14, pp. 180-186, 1992.
[21] H. M. Lankarani, P.E. Nikravesh, "Continuous contact force models for impact analysis in multibody systems". *Nonlinear Dyn*. 5, pp.93–207, 1994.
[22] F. Pfeiffer, Glocker, C., *"Multibody Dynamics with Unilateral Contacts"*, Willey-VCH Verlag Gmbh&Co, KGaA, 2004.
[23] F. Pfeiffer, C. Glocker, *"Multibody Dynamics with Unilateral Contacts"* (Wiley Series Nonlinear Science), Wiley-VCH, 1996.
[24] E. T. Routh, *"Dynamics of a system of rigid bodies"*. Macmillan, London, 1905.
[25] A. A. Shabana "*Dynamics of Multibody Systems"*. 3 rd Ed. Cambridge University Press, New York, 2010.
[26] W. J. Stronge, "*Impact Mechanics*" Cambridge University Press, Cambridge, 2000.
[27] W. J. Stronge, James R., Ravani, B., "Oblique Impact With Friction and Tangential Compliance", *Phil. Trans. R. Soc. Lond.* A, 359, 2447-2465, 2001.
[28] D. Tabor, Proc. R. Soc. Lond. A 229, pp.198-220, 1955.
[29] S. P. Timoshenko, J.N. Goodier, "*Theory of elasticity*". McGraw-Hill, New York., 1970;
[30] R. Voinea, D. Ceausu, V. Voiculescu, "*Mecanica"*, EDP Bucuresti, 1983.
[31] Wang Yu, "*On Impact Dynamics of Robotic Operations"*, Carnegie Carnegie-Mellon University Pittsburgh, 1986
[32] Y. Wang , M. T. Mason, "Two dimensional rigid-body collisions with friction" *Journal of Applied Mechanics* 59:635–642., 1992.
[33] E. T. Whittacker,"*A Treatise On the Analytical Dynamics of Particles and Rigid Bodies: With an Introduction to the Problem of Three Bodies";* FQ Books, 2010.
[34] J. Wittenburg, "*Dynamics of Multibody Systems"*, Publisher: Springer; 2nd ed. 2008.
[35] J. A. Zukas, T. Nicholas, L. B. Greszczuk, D. R. Curran, *"Impact Dynamics"*, Wiley, New York, 1982.
[36] P. E. Nikravesh, "*Computer-aided analysis of mechanical systems*", Prentice Hall, Englewood, 1988.
[37] R. M. Brach, "*Mechanical Impact Dynamics, Rigid Body Collisions"* , New York: Wiley, 1991.
[38] T. R. Kane, "A Dynamic Puzzle", *Stanford Mechanics Alumni Club Newsletter,* pp. 6-10, 1984.
[39] S. Alaci, F. C. Ciornei, „Comparative analysis of indentations from spatial impact with plastic deformations", *ANNALS OF THE ORADEA UNIVERSITY.Fascicle of Management and Technological Engineering*, Volume XIII (XXIII), NR1 May, pp. 107-110, 2014.
[40] F. C. Ciornei, S. Alaci, „Plastic Indentation Analysis Used in Study of Colliding Robotic Elements", *Robotica & Management*, 19/1, pp. 11-16, 2014.