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EQUIVALENCE BETWEEN BALL-ROTATING DISC IMPACT AND BALL-INCLINED PLANE IMPACT, PART II: ANGULAR VELOCITY COMPARISON

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Abstract—The condition of similarity between the spatial impact of ball-rotating disc and the plane impact of ball-inclined plane were précised in the first part of the paper and the concordance between the plastic indentations obtained in the two impact situations, when the relative motions between initial contacting points are the same, was validated. The present work presents an improved non contact method for estimating the post impact angular velocity of the ball. The results obtained for the values of angular velocity together with the values obtained for the values of centre of mass and reflection angle, in another work of the authors, show that despite ensuring the same values and direction of relative velocity, the post impact kinematical characteristics of the ball are different for the two cases.

Keywords— impact, angular velocity, high speed camera

I. Introduction

THE possibility of equivalence concerning the spatial impact occurring between a free falling ball onto a horizontal plane face of a metallic disc rotating around a vertical axis and the plane impact between the same ball colliding a fixed inclined plane is aimed, as shown in the first part of the paper. An affirmative response for this question would simplify noticeably the study of spatial collision because for the plane impact with friction, the literature offers an extremely simple and efficient method, the plane of percussions method, given by Routh. A correct correspondence should denote that ensuring before impact the same relative motion between the impacting points from the ball and disc, or ball and inclined plane, respectively, the post impact ball motion ought to be the same for both situations. To this restraint, the condition of identical post impact plastic imprints for the two collisions is added. The condition of the same ball motion for the two cases is expressed, according to Euler's equation [1] that describe the velocity distribution for a rigid body, as:

1) the same velocities for mass centre of the ball;
2) the same angular velocities of the ball.

The methodology and results are presented in [2].

The impact with friction is quite often met in technological applications, starting from the effect of clearances in kinematical joints and ending with contact initiation between a robotic hand and the manipulated object.

II. Finding the Angular Velocity of the Ball

The optical method was considered a good alternative in finding the post impact angular velocity of the ball, for minimum intrusion within the experimental results. The direction of post impact angular velocity is a first theme requiring an answer. Given that the final purpose of the analysis is to precise if the method proposed by Routh [3] for study of plane collisions with dry friction may be applied to the impact between a rotating disc and a ball, it must be verified that the direction of post impact angular velocity should be normal to the plane defined by the normal in the contact point and the relative impact velocity.

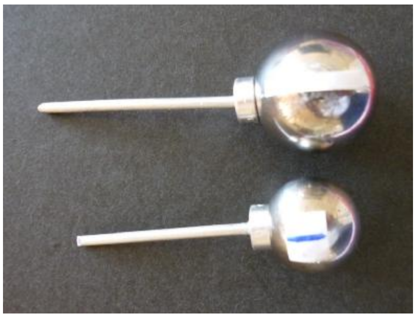


Fig. 1. Balls with attached rods and autoadhesive stamps

The motion the ball will have after the impact with the rotating disc was studied concerning this subject. To this end, based on the previous results, [4], to highlight the rotation motion of the ball, an adhesive stamp was applied together with a radial aluminum rod attached to the sphere. The ball was hold in a specially designed launcher by means of the rod. The ball was carefully oriented such that the mark should be in the closest plane to the camera, Fig. 1. Moreover, the direction the camera was positioned was set as shown in Fig. 2. The radial direction will be the direction of observation.

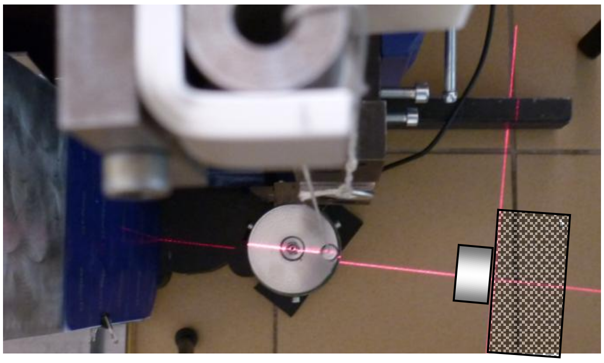
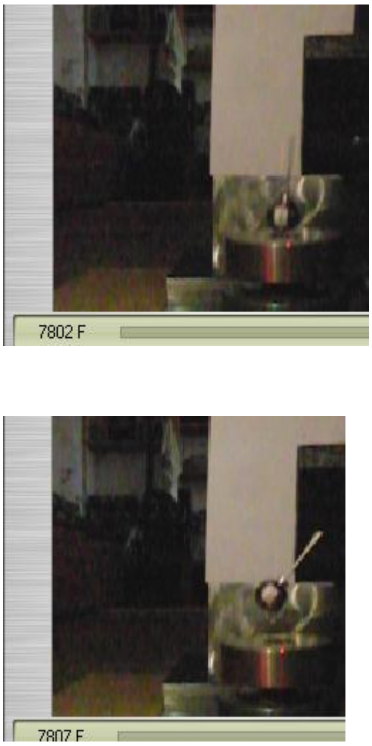


Fig. 2. Positioning of the video camera

From the two directions, radial and tangential, the radial direction was chosen for analysis since, under the assumption of plane post impact motion, it is required that the stamp should remain permanently parallel to the vertical plane in which it was contained at launching.

Fig. 3 presents a couple of images obtained by movie breaking into frames at 420 frames/sec.



a)

b)

Fig. 3. Two ball positions after the impacting the disc

Fig. 3 confirms that the angular velocity of the ball is normal to image plane and thus the ball-disc impact can be regarded as a plane impact. Additionally, taking into account that after impact the sum of external moments upon the ball is zero, [5]:

$$\sum \mathbf{M}_{\mathbf{ext}} = 0\,, \tag{1}$$

From moment of momentum theorem it results:

$$\boldsymbol{\omega} = \mathbf{const.} \tag{2}$$

Thus, the angle between the rod directions is found from two post impact images and by dividing it to the time period between the two images, the angular velocity after collision is obtained. The time between the two frames is easily found since the software has the option of indicating the frame index, as shown in Fig. 3.

Because the angular velocity must be found simultaneously with the velocity of the centre of mass, for the same impact, the velocity of centre of mass evaluation is based on measuring the time between the ball-disc impact and a second impact between the ball and a vertical metallic wall. But, as shown in Fig. 4, there is the possibility that the rod strikes the wall and not the ball,

Fig. 4. The case when the metallic plate is in contact with the rod

Fig. 5 presents the case when the ball has the rod detached for finding the angular velocity and Fig. 6 presents two positions of the ball with the rod attached. In both figures the first frame is randomly taken and the second frame is chosen thus as the ball rotation should be as close as possible to a complete rotation.

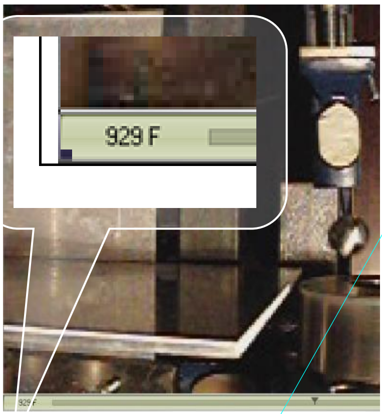
Knowing that the image capture velocity is 420frames/sec for Fig. 5, it results:

$$\boldsymbol{\omega} = \frac{\mathbf{2\pi}}{\mathbf{(970-929)}}\mathbf{420} = \mathbf{64.3 rad/sec}\,, \tag{3}$$

and for Fig. 6:

$$\boldsymbol{\omega} = \frac{\mathbf{4\pi}}{\mathbf{(1000-864)}}\mathbf{420} = \mathbf{38.8 rad/sec}\,. \tag{4}$$

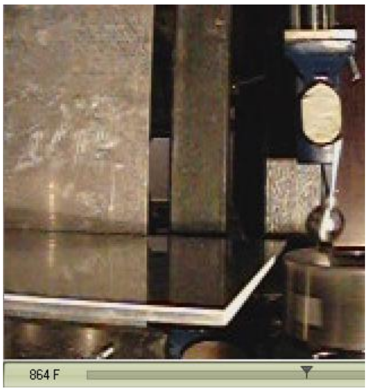
The great difference between the values of angular velocity for the two cases, ball and ball with rod shows the ball with attached rod must be abandoned. The frontal faces of the same cylinder were used in both collisions in order modify as less as possible the impact conditions. For accurate adjustment of the rig parts, a digital level meter was used (Fig. 7) for setting the tilting angle. In the case of rotating disc, the horizontality should be carefully controlled, as shown in Fig. 8, where a bearing ball is placed on the disc and the ball's equilibrium ensures horizontality more exact than the level meter accuracy.



a)

b)

Fig. 5. Angular velocity evaluation for ball without the rod



a)

b)

Fig. 6. Comparison between angular velocities from ball-disc impacts. Angular velocity evaluation for ball with the rod

Fig. 7. Adjusting the tilt angle of the plane

Fig. 8. Disc's deviation from horizontal plane test and contact point position identification

TABLE I
EXPERIMENTAL DATA

Disc rotation velocity n [rot/min]	**480**	**480**	**680**	**1500**
Inclined plane angle α' [deg]	24.1	24.1	32.4	**54.5**
Height of falling on inclined plane H [m]	0.89	0.89	1.04	**2.20**
Angular velocity, post impact *ball-disc* ω_b [rad/sec]	91	90.1	94.28	**39.94**
Angular velocity, post impact *ball-plane* ω'_b [rad/sec]	121	119.	114.7	-
Post impact velocity, ball- disc v''_0 [m/sec]	1.808	1.878	1.782	**1.737**
Launching angle post impact, ball-disc α''_0 [deg]	16.32	15.89	16.86	**9.22**
Velocity post impact ball-plane v''_0 [m/sec]	2.065	-	2.703	-
Launching angle post impact, ball-plane α''_0 [deg]	**39.93^0**	-	**27.532^0**	-

III. Experimental results

For the study of ball-inclined plane impact the adjustment of the device is required, specifically the tilt angle of the plane and the height of ball falling. These values were found from kinematical equivalence condition. The values of the tilting angle of the inclined plane and the launching height are given in Table I. The tests made were corresponding to rotation velocities $n = 480 rot / min$ and $n = 680 rot / min$. For the rotation velocity $n = 1500 rot / min$ the height of launching ought to be $H = 2.20m$, value that is not possible for the actual device. For tests with ball launchings corresponding to velocities greater than $700 rot / min$, a ball spring launcher must be executed.

Table I presents in synthesis the experimental parameters: the values of disc angular velocity, corresponding angle of inclined plane, the height of ball launching, the last two found from the condition of kinematical equivalence presented in the first part of the work. The last two lines present the angular velocities of the ball for the two impact types. For the disc-ball impact, the height of launching was $h = 0.74m$ and the radius of contact point $r = 0.035m$. One cannot consider that the differences are unimportant.

IV. Conclusions

The present paper analyses the possible similarity between the spatial collision free falling ball-rotating disc and the plane collision of the same ball with an inclined plane. An affirmative answer to this subject would greatly simplify the study of spatial collisions.

In modeling the dynamical behavior of a system, sudden constraints occur often, and the effects are noticeable both at the interface of contact regions and inside the materials. Stress modeling and optimization can be applied in various domains, from robotic elements [6], [7] to classical engineering applications, [8].

From qualitative point of view, the paper illustrates that after colliding the rotating disc, the ball takes a rotation motion around a horizontal axis and a necessary condition that after impact the ball should have a plane motion is fulfilled.

The quantitative analysis assumed two aspects. First, the conditions of identical relative motions between colliding bodies were identified. More explicit, it is required that the velocity and incidence angle for the two impacts must be the same. To satisfy these conditions, two test rigs were designed and constricted. For both devices, the height of ball free fall can be adjusted and for the first device, the disc rotation velocity can be measured and regulated while for the second device, the tilt angle of the inclined plane can be accurately adjusted. For both collisions the flight time of the ball between the impact instant and the second impact with aluminum plate is measured. Another measured parameter is the distance between the plastic indentations from ball-disc and ball-plane tests, on one side, and ball-aluminum plate on the other side. With these parameters, using own developed programs, the pot-impact velocity and the reflection angle can be found.

Another aspect envisaged during experimental researches consists in geometrical characteristics of post impact plastic indentations for the two impact cases. This analysis offers information upon the work of plastic deformation produced during the impact process.

As an outcome of the tests, one can conclude that:

1) following the impact, the velocity of the centre of the ball is smaller in the case of ball-disc impact;

2) the reflection angle is smaller in the case of ball-disc impact;

3) the tangential profiles are practical identical for the two values of angular velocities of the disc. Moreover, for the indentations corresponding to two collisions at the same angular velocity of the disc, there are very close, proving a good repeatability of the results.

From the above remarks one can say that the impact between a ball and a rotating disc cannot be equivalent to the impact between the same ball and an inclined plane.

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