OPTIMAL DESIGN OF QUADRATIC SANDWICH PLATE

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Abstract**—**In this paper we show the optimal design of the three-layered sandwich plates. The objective function contains the material and fabrication costs. The design constrains are the maximal stresses, the deflection of plates and damping of vibrations. The unknown is the thickness of the filling foam. By the mathematical method we define the minima of cost function and the optimal thickness of the filling layer of foam. The active constraint is the deflection, so we calculate of the costs of sandwich plate with the homogeneous plate.

Keywords**—**Optimal design, reduction of costs, sandwich plate.

I. Introduction

THE suitable application of the mathematical methods of optimal design can reach, that the different constructions, products not only satisfy the requirements with the technical requirements, but they would be economical, too. The tasks of technical optimizations can be divided into two groups: structural and technological optimization. The structural optimization could make a group of topological optimization, shape optimization, size optimization and material optimization [1]. With the spread of the modern plastic foams parallel the sandwich constructions manufacture is accelerated. We can distinguish three types of structures: beams, plates and shells. The constructions of sandwiches generally make three layers (rarely more). The typical method of three layers construction of sandwich is that the outside layers are the faces (h_1 and h_3) generally made from some kind of metal and the inside layer is the core (h_2) made from some kind of easy material e.g. plastic foam, as Fig. 1. shows it.

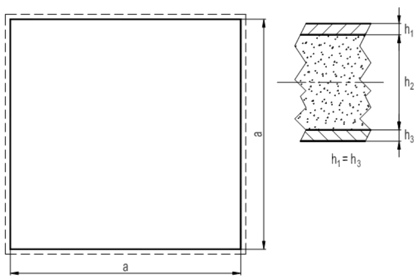

Fig. 1. The structure of sandwich plate

The facings are taking of normal stresses that come from the bending and compression. The core insures the working together of construction and takes up the nearly all shearing stresses.

II. The objective function and constraints

A. Formulation of objective function

The aim of optimization is the cost reduction of three layers sandwich plate fabrication that could be seen in Fig. 1. The faces made from alumina and steel (h_1=h_3=h) and the core made from polyurethane foam. We select unknown variable the thickness of foam (h_2). The objective function (K) consists of the following elements:

$$\mathbf{K = K_{al} + K_{cut} + K_c + K_{foam}}, \quad (1)$$

where K_{al} cost of alumina and steel that we calculate with the next form:

$$\mathbf{K_{al} = k_{al} V}, \quad (2)$$

where k_{al} is the specific cost of alumina and steel. V is the volume of iron plates. K_{cut} is the cost of cutting:

$$\mathbf{K_{cut} = \sum_{i=1}^{n} k_{cut} l_{lv}}, \quad (3)$$

where k_{cut} is the specific cutting cost and l_{vi} is the long of i-th cutting. K_t is the surface cleaning cost:

$$\mathbf{K_c = k_c A_c}, \quad (4)$$

where k_c is the specific cleaning cost and A_c is the surface. At the sandwich constructions could be done by gluing or foaming fixing the core. Of late years the foaming is generally applied, so in our faculty we used this method, too. We calculated the cost of foaming by this form [2]:

$$\mathbf{K_{foam} = (t_e + \frac{a^2 h_2}{V_h} + t_{ki})(k_m + k_a) + a^2 h_2 k_{foam}}, \quad (5)$$

where t_e is the time of preparation, V_h is the volume flow rate of foam, t_{ki} is the time of foaming, k_m is the specific coast of wage, k_a is the coast of amortization of fittings and f_{foam} is the specific cost of foam.

B. Formulation of constraints

1. *The constrains for the maximal deflection*

At the sandwich plates the shearing change of form is considerable, so we have to take care it. By the permanent split lateral loading have to be the maximal deflection than the allowable (w_{all}) [3]- [4]:

$$w_{max} = \frac{pa^4}{D_2}(\beta_1 + \rho\beta_2) \le w_{all}, \quad (6)$$

where p is the loading and

$$D_2 = \frac{Eh(h_2+h)^2}{2(1-\nu^2)}, \qquad \rho = \frac{\pi^2 Eh(h_2+h)}{2(1-\nu^2)G_2 a^2}, \quad (7)$$

$$\beta_1 = \sum\sum \frac{16}{\pi^4}\frac{(-1)^{(m-1)/2}(-1)^{(n-1)/2}}{mn\Omega^2}, \quad (m,n-odd), \quad (8)$$

$$\beta_2 = \sum\sum \frac{16}{\pi^4}\frac{(-1)^{(m-1)/2}(-1)^{(n-1)/2}}{mn\Omega}, \quad (m,n-odd), \quad (9)$$

where $\Omega=m^2+n^2$, ν is the Poisson's ratio, E is the modulus of elasticity of facings and G_2 is the shear modulus of core.

2. *The constraint for the maximal normal stress in the facings*

We can calculate the maximal normal stresses with the next formula and its values have to be smaller than the allowable:

$$\sigma_{max} = \frac{pa^2}{(h_2+h)h}\beta_3(1+\nu) \le \sigma_{all}, \quad (10)$$

where

$$\beta_3 = \sum\sum \frac{16}{\pi^4}\frac{(-1)^{(m-1)/2}(-1)^{(n-1)/2}}{\Omega^2}. \quad (11)$$

3. *The constraint for the maximal shear stress in the facings*

We can calculate the maximal shear stresses with the next formula and its values have to be smaller than the allowable for the material of facings:

$$\tau_{hmax} = \frac{pa^2}{(h_2+h)h}(1-\nu)\beta_5 \le \tau_{hall}, \quad (12)$$

where

$$\beta_5 = \sum\sum \frac{16}{\pi^4}\frac{1}{\Omega^2}. \quad (13)$$

4. *The constraint for the maximal shear stress in the core*

It is typical at the sandwich construction that the gross of shear recourse the core takes up. In the core we calculate the maximal shear stresses with the next formula [2-3]:

$$\tau_{2max} = \frac{pa}{(h_2+h)}\beta_6 \le \tau_{2all}, \quad (14)$$

where

$$\beta_6 = \sum\sum \frac{16}{\pi^3}\frac{(-1)^{(n-1)/2}}{n\Omega}. \quad (15)$$

5. *The constraint for the loss factor of faces*

The loss factors of metals are very small. Using of good damping materials at the sandwich constructions we can significantly increase this factor. To measure and calculate of vibration damping are several methods. In the [5] literature there is a connection that modified we could calculate the loss factor of sandwich plate (η) connection with the geometrical sizes and quality of materials. We prescribe that the loss factor of sandwich structure would be some percentages of damping factor of foam of polyurethane:

$$c^*\eta_2 \le \eta = (Q_2R_1 - Q_1R_1)/(Q_1R_1 + Q_2R_2), \quad (16)$$

where

$$Q_1 = \frac{8\pi^4}{(1-\nu^2)}\left[\frac{G_{2d}(1-\nu^2)}{}\frac{16h^3+24h^2h_2+12hh_2^2}{3Ea^3} + \frac{8h^4h_2\pi^2}{3a^5}\right], \quad (17)$$

$$Q_2 = \frac{32\pi^4 G_{2d}(1-\nu^2)(16h^3+6h^2h_2+3hh_2^2}{3Ea^3}, \quad (18)$$

$$R_1 = \frac{4E\pi^2hh_2 + 2G_{2d}a^2(1-\nu^2)}{Ea^2}, \quad (19)$$

$$R_2 = \frac{2G_{2d}\eta_2(1-\nu^2)}{E}, \quad (20)$$

where $c^*$is the prescribed loss factor of sandwich plate and G_{2d} is the dynamics shear modulus of the core.

6. *Geometrical constraints*

In many cases constraining of the sizes of structures is required. We build into optimization model that restrict the minimal or maximal value of geometrical sizes. In this case we restrict the height of core:

$$h_{2min} \le h_2 \le h_{2max}. \quad (21)$$

7. *Constraint of homogenous plate maximal deflection*

At the optimization of sandwich plate the deflection constraint is active. For this reason we compare the costs of homogenous plate with sandwich plate by some deflection. We use the function of simply supported sandwich plate with uniformly distributed, lateral load [6]:

$$w_0 = c_1\frac{pa^4}{Et^3}, \quad (22)$$

where c_1=0.04464 and t is the thickness of homogenous plate.

III. NUMERICAL DATA AND RESULTS

In the Fig. 1. could be seen simply supported sandwich plate where data are: a =1000, 1500 (mm); k_{al}=4,53x10^6 (HUF/m^3); k_{cut} =350 (HUF/m); k_c =200 (HUF/m^2); k_m=40 (HUF/min); k_a=0,316: β_1=0,0042; β_2=0,0077; β_3=β_4=0,038; β_6=0,33; h=3mm; c^*=0,5; η_2=0,22; t_e=10 (min); t_{ki}=8 (min); V_h = 0,1 (m^3/min); t_{ki}=8 (min); k_{foam}

=11 000 (HUF/m^3); E= 210 (GPa) (steel), E=70 (GPa) (alumina); G_2=3,1 (MPa); G_{2d}=0,69 (MPa); σ_{all}=115 (MPa); τ_{hall}=30 (MPa); w_{all}=3 (mm); ν=0,33, τ_{2all}=0,2 (MPa); τ_{hall}=30 (MPa); η_2=0,22; h_{2min}=20 (mm); h_{2max}=350 (mm).

After calculations there are the results:

TABLE I
SANDWICH PLATE WITH ALUMINA FACING

Load p(N/mm^2)	h_2(mm)	K_{min} (HUF)
0,010	80	30 620
0,015	121	31 086
0,020	162	31 552
0,025	203	32 017
0,030	244	32 483
0,035	284	32 949
0,040	325	33 415

a = 1000 mm and material is alumina.

TABLE II
SANDWICH PLATE WITH ALUMINA FACING

Load p(N/mm^2)	h_2(mm)	K_{min} (HUF)
0,004	74	66 771
0,006	110	67 716
0,008	147	68 660
0,010	184	69 604
0,012	220	70 547
0,014	258	71 490
0,016	294	72 435
0,018	331	73 378

a = 1500 mm and material is alumina.

TABLE III
SANDWICH PLATE WITH STEEL FACING

Load p(N/mm^2)	h_2(mm)	K_{min} (HUF)
0,010	82	5881
0,015	123	6347
0,020	164	6813
0,025	205	7279
0,030	246	7745
0,035	287	8211
0,040	327	8677

a = 1000 mm and material is steel.

TABLE IV
SANDWICH PLATE WITH STEEL FACING

Load p(N/mm^2)	h_2(mm)	K_{min} (HUF)
0,004	75,8	11 112
0,006	113	12 056
0,008	149	13 001
0,010	186	13 944
0,012	223	14 888
0,014	260	15 831
0,016	297	16 775
0,018	333	17 719

a = 1500 mm and material is steel.

In the tables (below) on the left side we see the results of sandwich plates (I-IV tables) and on the right side are the results of homogenous plate (V-VIII tables). From the results we can read that in case of homogenous plate more material of metal is needed to the same deflection. So the costs are higher than sandwich plates.

TABLE V
HOMOGENOUS PLATE FROM ALUMINA

Load p(N/mm^2)	h(mm)	K_{min} (HUF)
0,010	12,86	58 256
0,015	14,71	66 636
0,020	16,20	73 386
0,025	17,45	79 049
0,030	18,54	83 986
0,035	19,52	88 426
0,040	20,41	92 457

a = 1000 mm and material is alumina.

TABLE VI
HOMOGENOUS PLATE FROM ALUMINA

Load p(N/mm^2)	h(mm)	K_{min} (HUF)
0,004	16,26	165 730
0,006	18,62	189 784
0,008	20,50	208 946
0,010	22,07	224 948
0,012	23,46	239 104
0,014	24,70	251 829
0,016	25,82	263 170
0,018	26,86	273 771

a = 1500 mm and material is alumina.

TABLE VII
HOMOGENOUS PLATE FROM STEEL

Load p(N/mm^2)	**h(mm)**	**K_{min} (HUF)**
0,010	**5,25**	**5138**
0,015	**5,15**	**6226**
0,020	**5,95**	**7194**
0,025	**6,65**	**8040**
0,030	**7,29**	**8814**
0,035	**7,89**	**9539**
0,040	**8,41**	**10168**

a = 1000 mm and material is steel.

TABLE VIII
HOMOGENOUS PLATE FROM STEEL

Load p(N/mm^2)	h(mm)	K_{min} (HUF)
0,004	11,46	31 174
0,006	13,12	35 690
0,008	14,44	39 280
0,010	15,56	42 327
0,012	16,53	44 966
0,014	17,41	47 360
0,016	18,20	49 509
0,018	18,93	51 494

a = 1500 mm and material is steel.

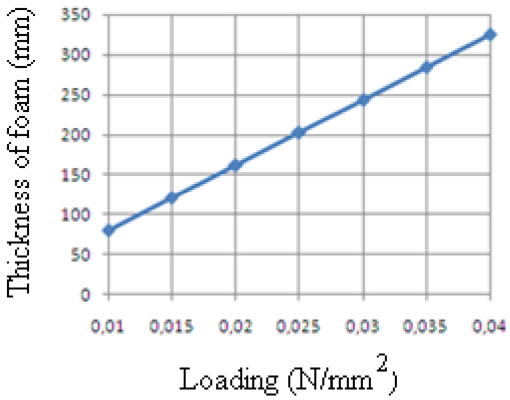

Fig. 2. Thickness of foam in case of alumina and a= 1000 mm

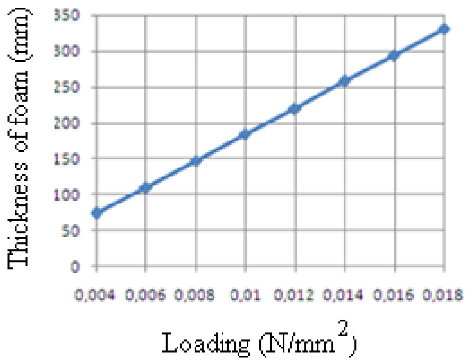

Fig. 3. Thickness of foam in case of alumina and a= 1500 mm

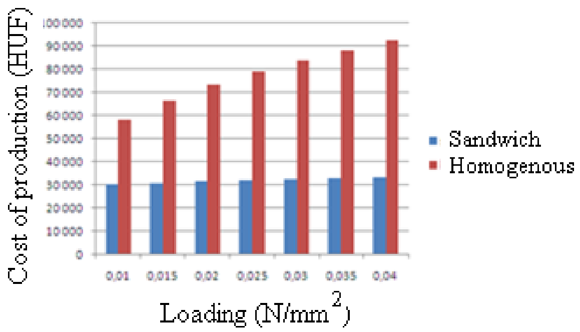

Fig. 4. Costs of sandwich and homogenous plates in case of alumina and a= 1000 mm

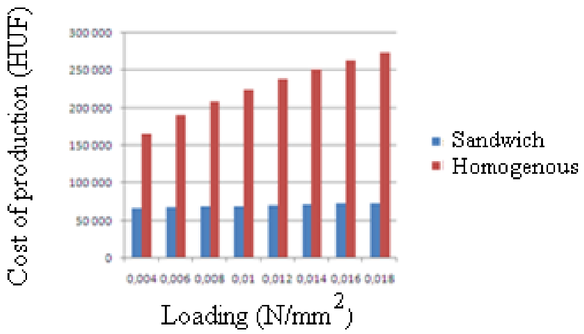

Fig. 5. Costs of sandwich and homogenous plates in case of alumina and a= 1500 mm

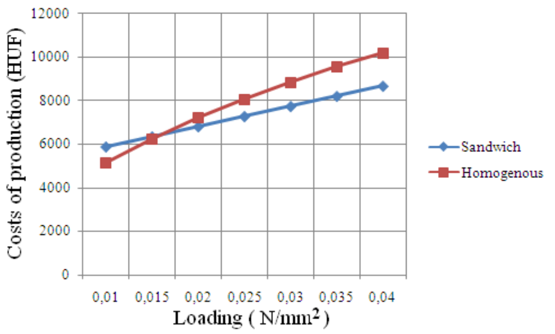

Fig. 6. Costs of sandwich and homogenous plates in case of steel and a= 1000 mm

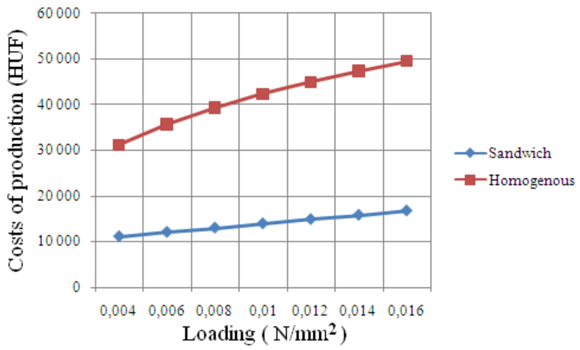

Fig. 7. Costs of sandwich and homogenous plates in case of steel and a= 1500 mm

From the advantageous mechanical behavior of sandwich plate derive that the faces are far away from the bending axis so its bending stiffness is higher than the homogenous plate. It is the reason why sandwich plate is cheaper than the homogenous plate.

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