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MATHEMATICAL MODEL FOR DETERMINING THE EFFORTS FROM CARDAN JOINT MECHANISM WITHOUT TECHNICAL DEVIATIONS

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Abstract—The cardan joint mechanism enables the transmission of the rotation movement between two shaft trees which form a constant or variable angle in time. This mechanism is of the third family , as a consequence, multiple statically indeterminate and for calculating the reactions from the kinematic pairs it is applied the elastic linear calculation using the relative displacements method noted in plücker coordinates. In this paper are deducted the calculation relations and is established the mathematical model for determining the efforts of the ideal (without technical deviations) cardan joint mechanism, starting from the RRRR spatial quadrilateral mechanism.The results of the numerical solving of this problem will be presented under the form of a diagram and will be commented.

Keywords—cardan joint, cardan transmission, elastic calculation.

I. INTRODUCTION

BEING statically undetermined [1]-[4], for the 4R spherical quadrilateral mechanism is necessary to be used the elastic linear calculation for calculating the reactions.

The obtained results allow us to conclude the influence of geometrical and mechanical parameters over the unitary efforts that appear in the kinematic pairs of the cardan joint mechanism.

II. GENERAL ASPECTS

The mechanically speaking (static) , the 4R mechanism has unknown the reactions from the kinematic pairs A, 2, 3, 5, 6, 8 and also the moment from the joint A (see Fig. 1.), in total 31 unknowns and 18 equations, result in 13 times statically undetermined.

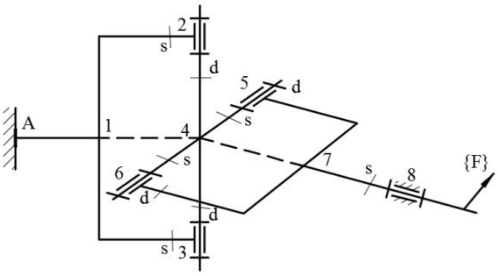

Fig. 1. Symmetrical Spherical Quadrilateral Mechanism.

For determining these components a linear elastic calculation is used.In the following, a mathematical model with being elaborated for the linear elastic calculation of reactions, a model that has as basis the method of relative displacements, presented in paper [5], with the notation in plücker coordinates.

In the elastic calculation the joint from A is blocked (see Fig. 1.), the thing that explains the apparition as unknown of the axial moment at that point.

The pivotal points 1, 4, 7, have the displacements $\{\Delta_1\}$, $\{\Delta_4\}$, $\{\Delta_7\}$, and the pivotal points with the cinematic couples 2, 3, 5, 6 have left-right displacements with $\{\Delta_2^s\}$, $\{\Delta_2^d\}$, $\{\Delta_3^s\}$, $\{\Delta_3^d\}$, $\{\Delta_5^s\}$, $\{\Delta_5^d\}$, $\{\Delta_6^s\}$, $\{\Delta_6^d\}$.

Pivotal point 8 has the displacement to the left $\{\Delta_8^s\}$ and to the right $\{\Delta_8^d\}=\{0\}$.

So the relations are written under the form:

$$\begin{cases} \{\Delta_2^d\}=\{\Delta_2^s\}+\xi_2\{U_2\} \\ \{\Delta_3^d\}=\{\Delta_3^s\}+\xi_3\{U_3\} \\ \{\Delta_5^d\}=\{\Delta_5^s\}+\xi_5\{U_5\} \\ \{\Delta_6^d\}=\{\Delta_6^s\}+\xi_6\{U_6\} \\ \{0\}=\{\Delta_8^s\}+\xi_7\{U_7\}. \end{cases} \quad (1)$$

From the pivotal points balance results

$$\begin{cases} [K_{12}]\left(\{\Delta_1\}-\{\Delta_2^s\}\right)+[K_{1A}]\left(\{\Delta_1\}-\{\Delta_A^0\}\right)+ \\ \qquad [K_{13}]\left(\{\Delta_1\}-\{\Delta_3^s\}\right)=\{0\} \\ [K_{21}]\left(\{\Delta_2^s\}-\{\Delta_1\}\right)+[K_{24}]\left(\{\Delta_2^d\}-\{\Delta_4\}\right)=\{0\} \\ [K_{31}]\left(\{\Delta_3^s\}-\{\Delta_1\}\right)+[K_{34}]\left(\{\Delta_3^d\}-\{\Delta_4\}\right)=\{0\} \\ [K_{42}]\left(\{\Delta_4\}-\{\Delta_2^d\}\right)+[K_{43}]\left(\{\Delta_4\}-\{\Delta_3^d\}\right)+ \\ +[K_{45}]\left(\{\Delta_4\}-\{\Delta_5^s\}\right)+[K_{46}]\left(\{\Delta_4\}-\{\Delta_6^s\}\right)=\{0\} \\ [K_{54}]\left(\{\Delta_5^s\}-\{\Delta_4\}\right)+[K_{57}]\left(\{\Delta_5^d\}-\{\Delta_7\}\right)=\{0\} \\ [K_{64}]\left(\{\Delta_6^s\}-\{\Delta_4\}\right)+[K_{67}]\left(\{\Delta_6^d\}-\{\Delta_7\}\right)=\{0\} \\ [K_{75}]\left(\{\Delta_7\}-\{\Delta_5^d\}\right)+[K_{76}]\left(\{\Delta_7\}-\{\Delta_6^d\}\right)+ \\ \qquad +[K_{78}]\left(\{\Delta_7\}-\{\Delta_8^s\}\right)=\{0\}. \end{cases} \tag{2}$$

$[Kij]=[Kji]$, where $[Kij]$, is the rigidity matrix of the *ij* element.Are noted

$$\begin{aligned} &[K_{11}]=[K_{1A}]+[K_{12}]+[K_{13}] \\ &[K_{22}]=[K_{21}]+[K_{24}] \\ &[K_{33}]=[K_{31}]+[K_{34}]\ ; \\ &[K_{44}]=[K_{42}]+[K_{43}]+[K_{45}]+[K_{46}] \\ &[K_{55}]=[K_{54}]+[K_{57}]\ ;\ [K_{66}]=[K_{64}]+[K_{67}] \\ &[K_{77}]=[K_{75}]+[K_{76}]+[K_{78}]. \end{aligned} \tag{3}$$

and taking into account the relations (1) it results

$$\begin{cases} [K_{11}]\{\Delta_1\}-[K_{12}]\{\Delta_2^s\}-[K_{13}]\{\Delta_3^s\}=\{0\} \\ [K_{21}]\{\Delta_1\}+[K_{22}]\{\Delta_2^s\}-[K_{24}]\{\Delta_4\}+ \\ \qquad +\xi_2[K_{24}]\{U_2\}=\{0\} \\ -[K_{31}]\{\Delta_1\}+[K_{33}]\{\Delta_3^s\}-[K_{34}]\{\Delta_4\}+ \\ \qquad +\xi_3[K_{34}]\{U_3\}=\{0\} \\ [K_{31}]\{\Delta_1\}+[K_{33}]\{\Delta_3^s\}-[K_{34}]\{\Delta_4\}+ \\ \qquad +\xi_3[K_{34}]\{U_3\}=\{0\} \\ -[K_{54}]\{\Delta_4\}-[K_{55}]\{\Delta_5^s\}-[K_{57}]\{\Delta_7\}+ \\ \qquad +\xi_5[K_{57}]\{U_5\}=\{0\} \\ -[K_{64}]\{\Delta_4\}+[K_{66}]\{\Delta_6^s\}-[K_{67}]\{\Delta_7\}+ \\ \qquad +\xi_6[K_{67}]\{U_6\}=\{0\} \\ [K_{75}]\{\Delta_5^s\}-[K_{76}]\{\Delta_6^s\}+[K_{77}]\{\Delta_7\}-\xi_5[K_{75}]\{U_5\}- \\ \qquad -\xi_6[K_{76}]\{U_6\}+\xi_8[K_{78}]\{U_8\}=\{0\}. \end{cases} \tag{4}$$

The notations are made as

$$\{\Delta\}=\left[\{\Delta_1\}^T,\{\Delta_2^s\}^T,\{\Delta_3^s\}^T,\{\Delta_4\}^T,\{\Delta_5^s\}^T,\{\Delta_6^s\}^T,\{\Delta_7\}^T\right]^T$$

$$\{\xi\}=[\xi_2,\xi_3,\xi_5,\xi_6,\xi_8]^T \tag{5}$$

$$[V_2]=[1,0,0,0,0]\ ;\ [V_3]=[0,1,0,0,0]\ ;\ [V_5]=[0,0,1,0,0]$$

$$[V_6]=[0,0,0,1,0]\ ;\ [V_8]=[0,0,0,0,1].$$

ξ_i , $[\xi]$ being the elastic rotations respectively the elastic rotations matrix from the kinematic pairs [5],[6] and $[V_i]$ the matrix that verifies the relation

$$\xi_i=[V_i]\{\xi\}. \tag{6}$$

and $\xi_2[K_{24}]\{U_2\}=[K_{24}]\{U_2\}[V_2]\{\xi\}$, and the analogue. $[U_i]$ being the column matrixes of the plücker coordinates attached to the kinematic pairs [5],[6].

Also, the notations are made

$$[K_1]=\begin{bmatrix} [K_{11}] & -[K_{12}] & -[K_{13}] & [0] & [0] & [0] & [0] \\ -[K_{21}] & [K_{22}] & [0] & -[K_{24}] & [0] & [0] & [0] \\ -[K_{31}] & [0] & [K_{33}] & [K_{34}] & [0] & [0] & [0] \\ [0] & -[K_{42}] & -[K_{43}] & [K_{44}] & -[K_{45}] & -[K_{46}] & [0] \\ [0] & [0] & -[K_{54}] & [0] & [K_{55}] & [0] & -[K_{57}] \\ [0] & [0] & -[K_{64}] & [0] & [0] & [K_{66}] & -[K_{67}] \\ [0] & [0] & [0] & [0] & -[K_{75}] & -[K_{76}] & [K_{77}] \end{bmatrix}. \tag{7}$$

$$[\mathbf{K}_2]=\begin{bmatrix}[\mathbf{0}]\\ [\mathbf{K}_{24}]\{\mathbf{U}_2\}[\mathbf{V}_2]\\ [\mathbf{K}_{34}]\{\mathbf{U}_3\}[\mathbf{V}_3]\\ -[\mathbf{K}_{24}]\{\mathbf{U}_2\}[\mathbf{V}_2]-[\mathbf{K}_{34}]\{\mathbf{U}_3\}[\mathbf{V}_3]\\ [\mathbf{K}_{57}]\{\mathbf{U}_5\}[\mathbf{V}_5]\\ [\mathbf{K}_{67}]\{\mathbf{U}_6\}[\mathbf{V}_6]\\ -[\mathbf{K}_{57}]\{\mathbf{U}_5\}[\mathbf{V}_5]-[\mathbf{K}_{67}]\{\mathbf{U}_6\}[\mathbf{V}_6]+\\ +[\mathbf{K}_{78}]\{\mathbf{U}_8\}[\mathbf{V}_8]\end{bmatrix}. \tag{8}$$

and then the equations (4) are combined into the equation

$$[K_1]\{\Delta\}+[K_2]\{\xi\}=\{\mathbf{0}\}. \tag{9}$$

equivalent with 42 scalar equations.

Isolating the left side of the pair 2, (see Fig. 1.) results that

$$\{\mathbf{R}_2\}=\{\mathbf{E}_2^s\}=[\mathbf{K}_{21}]\{\{\Delta_2^s\}-\{\Delta_1\}\}. \tag{10}$$

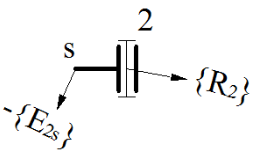

Fig. 2. The isolation of couple 2.

$[R_i]$, $\{E_i^s\}$ being the column matrixes of the plucker coordinates of the reactions from the kinematic pairs [5], [6] respectively the column matrixes of the efforts from the bar from the left of the kinematic pairs. And the analogue

$$\begin{cases}\{\mathbf{R}_3\}=\{\mathbf{E}_3^s\}=[\mathbf{K}_{31}]\{\{\Delta_3^s\}-\{\Delta_1\}\}\\ \{\mathbf{R}_5\}=\{\mathbf{E}_5^s\}=[\mathbf{K}_{54}]\{\{\Delta_5^s\}-\{\Delta_4\}\}\\ \{\mathbf{R}_6\}=\{\mathbf{E}_6^s\}=[\mathbf{K}_{64}]\{\{\Delta_6^s\}-\{\Delta_4\}\}\\ \{\mathbf{R}_8\}=\{\mathbf{E}_8^s\}-\{\mathbf{F}\}=[\mathbf{K}_{87}]\{\{\Delta_8^s\}-\{\Delta_7\}\}-\{\mathbf{F}\}.\end{cases} \tag{11}$$

$[F]$, being the column matrixes of the plücker coordinates of the external load.

As [5], $\{\tilde{U}_i\}^T\{R_i\}=0$ and $\{\Delta_8^s\}=-\xi_8\{U_8\}$,the equations are obtained

$$\begin{cases}\{\tilde{U}_2\}^T[K_{21}]\{\Delta_2^s\}-\{\tilde{U}_2\}^T[K_{21}]\{\Delta_1\}=\mathbf{0}\\ \{\tilde{U}_3\}^T[K_{31}]\{\Delta_3^s\}-\{\tilde{U}_3\}^T[K_{31}]\{\Delta_1\}=\mathbf{0}\\ \{\tilde{U}_5\}^T[K_{54}]\{\Delta_5^s\}-\{\tilde{U}_5\}^T[K_{54}]\{\Delta_4\}=\mathbf{0}\\ \{\tilde{U}_6\}^T[K_{64}]\{\Delta_6^s\}-\{\tilde{U}_6\}^T[K_{64}]\{\Delta_4\}=\mathbf{0}\\ \{\tilde{U}_8\}^T[K_{87}]\{\Delta_7\}+\xi_8\{\tilde{U}_8\}^T[K_{87}]\{U_8\}=\{\tilde{U}_8\}^T[F].\end{cases} \tag{12}$$

With the notations

$$[K_3]=\begin{bmatrix}\{\tilde{U}_2\}^T[K_{21}] & -\{\tilde{U}_2\}^T[K_{21}] & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \{\tilde{U}_3\}^T[K_{31}] & \mathbf{0} & -\{\tilde{U}_3\}^T[K_{31}] & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \{\tilde{U}_5\}^T[K_{54}] & -\{\tilde{U}_5\}^T[K_{54}] & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \{\tilde{U}_6\}^T[K_{64}] & \mathbf{0} & -\{\tilde{U}_6\}^T[K_{64}] & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \{\tilde{U}_8\}^T[K_{78}]\end{bmatrix}. \tag{13}$$

$$[\mathbf{K}_4]=\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}&\{\tilde{\mathbf{U}}_8\}^T[\mathbf{K}_{78}]\{\mathbf{U}_8\}\end{bmatrix};\{\tilde{\mathbf{F}}\}=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \{\tilde{\mathbf{U}}_8\}^T[\mathbf{F}]\end{bmatrix}. \tag{14}$$

the equations (11) are combined in the matrix equation

$$[\mathbf{K}_3]\{\Delta\}+[\mathbf{K}_4]\{\xi\}=\{\tilde{\mathbf{F}}\}. \tag{15}$$

equivalent with 5 scalar equations.

The equations (7), (15) can be narrowed with the notations

$$[\mathbf{K}]=\begin{bmatrix}[\mathbf{K}_1] & [\mathbf{K}_2]\\ [\mathbf{K}_3] & [\mathbf{K}_4]\end{bmatrix}. \tag{16}$$

in the equation

$$[\mathbf{K}]\begin{bmatrix}\{\Delta\}\\ \{\xi\}\end{bmatrix}=\begin{bmatrix}\{\mathbf{0}\}\\ \{\tilde{\mathbf{F}}\}\end{bmatrix}. \tag{17}$$

equivalent with 47 equations with 47 unknowns from which results $\{\Delta\}$ and $\{\xi\}$, and the reactions and efforts are calculated with the relations (10), (11) in which $\{\Delta_8^s\} = -\xi_8\{U_8\}$.

III. NUMERICAL APPLICATION

It is considered the cardan joint mechanism from Fig. 3.The mechanism's elements have the following geometrical and mechanical characteristics: the lengths $l_i = l = 0{,}06[m]$, the diameters $d_i = d = 0{,}02[m]$, the sections A_i, the elasticity modules $E = 2{,}1 \cdot 10^{11}\left[N/m^2\right]$, $G = 8{,}1 \cdot 10^{10}\left[N/m^2\right]$, and the main central inertial moments I_{iy}, I_{iz}, $I_{ix} = I_{iy} + I_{iz}$, $i = 1, 2, \ldots, 14$.The mechanism is driven by torque $\tilde{M} = 1[N \cdot m]$.

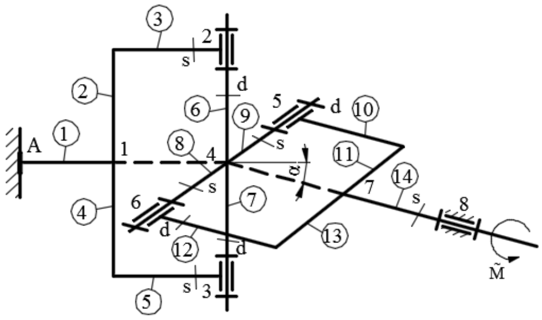

Fig. 3. Indexing bars.

The solving the mathematical model can be done using any conventional method of calculation numerical [7]. On the basis the mathematical model presented in this paper and the algorithm which will be presented in the next paper, has been realized in Excel a calculation program of the reactions from the kinematic pairs of the mechanism.As a result of numerical simulations were obtained the following results.

In the case where the angle between the cardan shafts is $\alpha = 0^\circ$, the reaction forces and moments, $R_{AX}^0, R_{AY}^0, R_{AZ}^0$, $M_{AX}^0, M_{AY}^0, M_{AZ}^0$ from the kinematic pair A, in the general system of reference varies as shown in Fig. 4. and Fig. 5. and in the local system of reference varies as shown in Fig. 6. and Fig. 7.

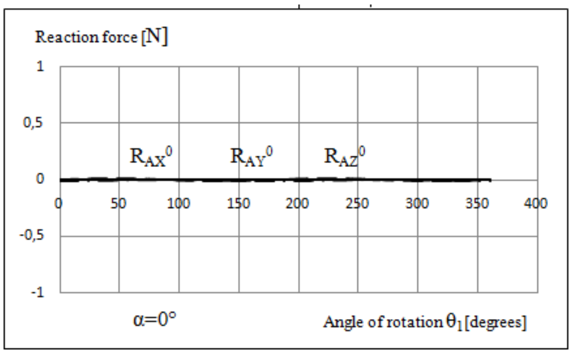

Fig. 4. The variation diagrams of reaction forces in the general reference system in kinematic pair A, for *α=0°*.

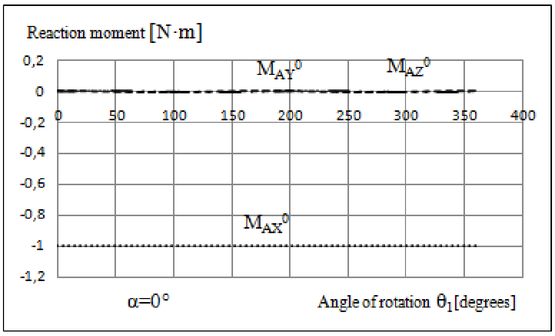

Fig. 5. The variation diagrams of reaction moments in the general reference system in the kinematic pair A, for *α=0°*.

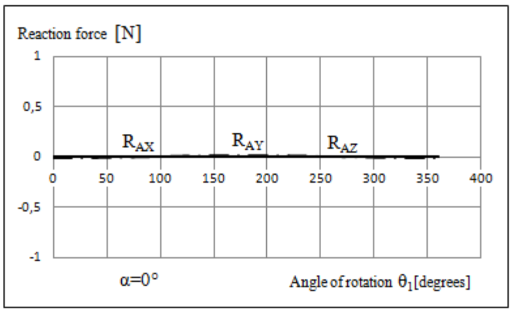

Fig. 6. The variation diagrams of reaction forces in the local reference system in the kinematic pair A, for *α=0°*.

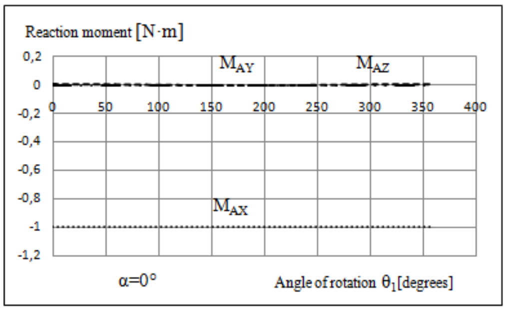

Fig. 7. The variation diagrams of reaction moments in the local reference system in the kinematic pair A, for *α=0°*.

For the reaction from the kinematic pair 2, in general reference systems, are obtained the values presented in Fig. 8. and Fig. 9. and in own reference systems values presented in Fig. 10. and Fig. 11.

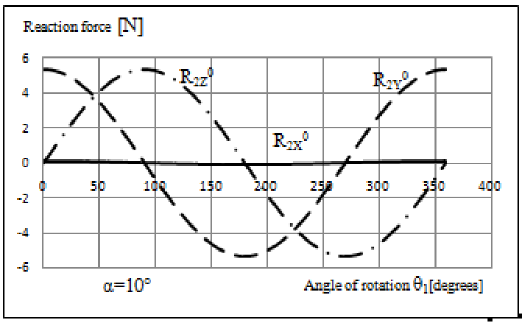

Fig. 8. The variation diagrams of reaction forces in the general reference system in the kinematic pair 2, for *α=0°*.

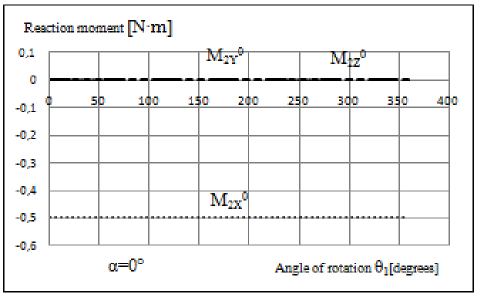

Fig. 9. The variation diagrams of reaction moments in the general reference system in the kinematic pair 2, for *α=0°*.

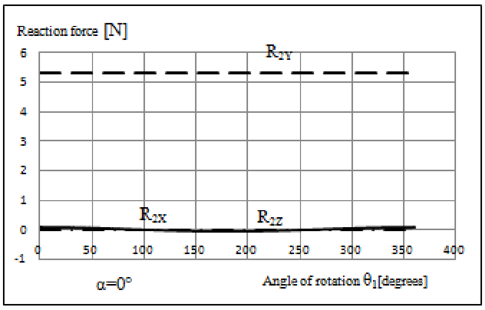

Fig. 10. The variation diagrams of reaction forces in the local reference system in the kinematic pair 2, for *α=0°*.

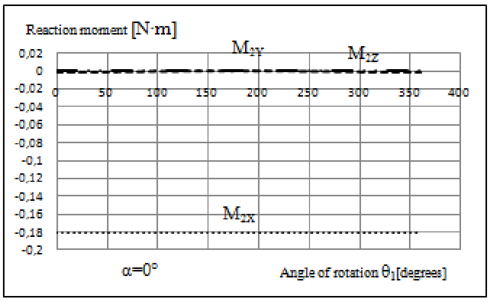

Fig. 11. The variation diagrams of reaction moments in the local reference system in the kinematic pair 2, for *α=0°*.

From the variation diagrams it is found that the reaction forces and moments are constant in the local systems of reference for all kinematic pairs.For the case where the angle between cardan shafts has $\alpha = 10^{\circ}$ have obtained the reactions $R_{AX}^{0}, R_{AY}^{0}, R_{AZ}^{0}$, M_{AX}^{0}, M_{AY}^{0}, M_{AZ}^{0} which varies as shown in Fig. 12. and Fig. 13.

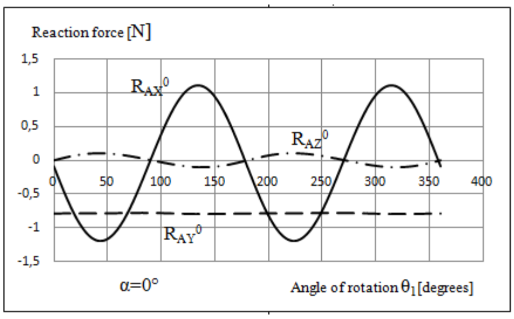

Fig. 12. The variation diagrams of reaction forces in the general reference system in the kinematic pair A, for *α=10°*.

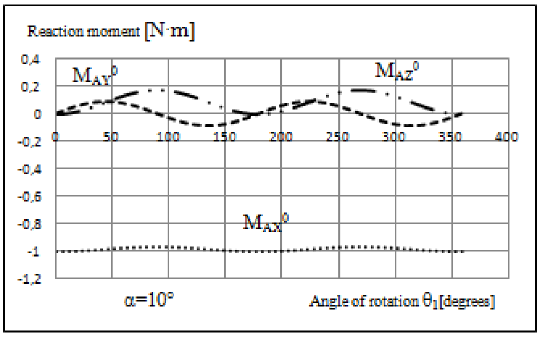

Fig. 13. The variation diagrams of reaction moments in the general reference system in the kinematic pair A, for *α=10°*.

From the diagrams of variation it is noticed that the components $R_{AX}^{0}, R_{AY}^{0}, R_{AZ}^{0}$, $M_{AX}^{0}, M_{AY}^{0}, M_{AZ}^{0}$, in the system local of reference aren't constant.Their variation depends on the angle θ_1.In Fig. 14., Fig. 15., Fig. 16. and Fig. 17. are represented the variation of the reaction forces and moments $R_{AX}^{0}, R_{AY}^{0}, R_{AZ}^{0}$, $M_{AX}^{0}, M_{AY}^{0}, M_{AZ}^{0}$, in the local system of reference for $\alpha = 20^{\circ}, \alpha = 30^{\circ}$.

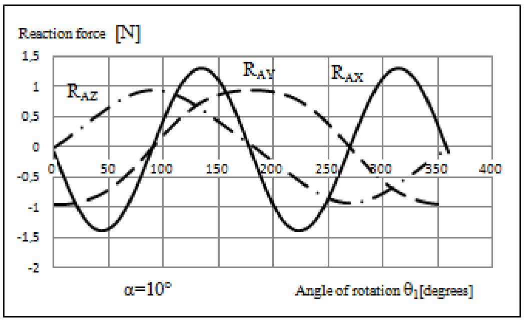

Fig. 14. The variation diagrams of reaction forces in the local reference system in the kinematic pair A, for *α=10°*..

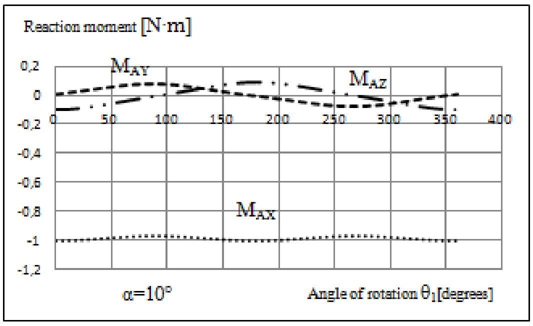

Fig. 15. The variation diagrams of reaction moments in the local reference system in the kinematic pair A, for *α=10°*.

Then, when the angle α increases, $\alpha = 20^{\circ}, \alpha = 30^{\circ}$, the solution for the components R_{AX}, R_{AY}, R_{AZ}, M_{AX}, M_{AY}, M_{AZ}, are represented in Fig. 16., Fig. 17., Fig. 18. and Fig. 19.

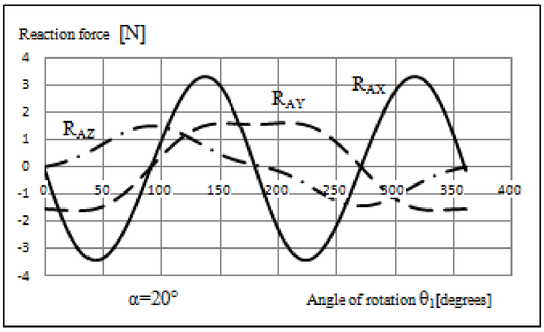

Fig. 16. The variation diagrams of reaction forces in the local reference system in the kinematic pair A, for *α=20°*..

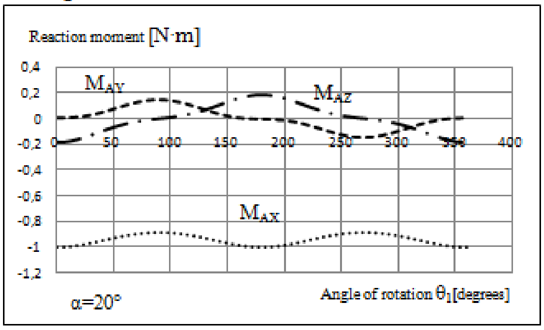

Fig. 17. The variation diagrams of reaction moments in the local reference system in the kinematic pair A, for *α=20°*.

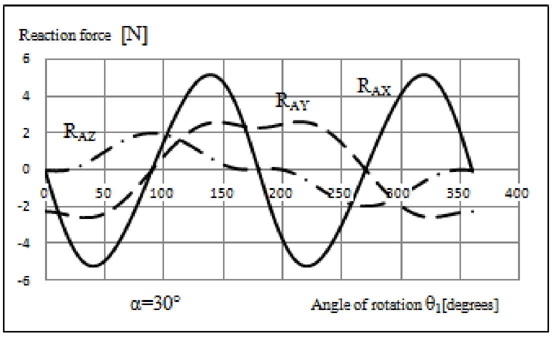

Fig. 18. The variation diagrams of reaction forces in the local reference system in the kinematic pair A, for *α=30°*..

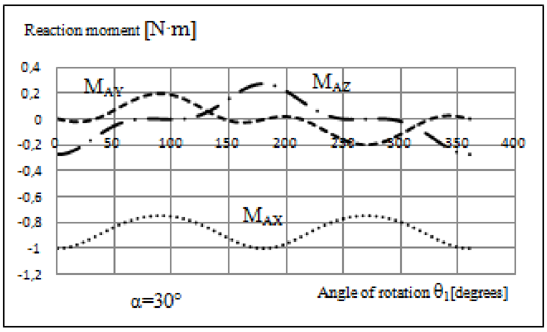

Fig. 19. The variation diagrams of reaction moments in the local reference system in the kinematic pair A, for *α=30°*.

The maxim value of the axial reaction R_{AX} from kinematic pairs A, increases from 0[N] ($\alpha = 0^\circ$) to 1,3[N] ($\alpha = 10^\circ$), to 3,2[N] ($\alpha = 20^\circ$) and to 5,3[N] ($\alpha = 30^\circ$), the reaction having also negative values.

IV. Conclusions

The elastic calculation and the relative displacements method makes possible the determination of the reactions from the kinematic pairs of the 4R spherical quadrilateral mechanism, that is multiple statically indeterminate.

Following the numerical simulations made with the program realized using the mathematical model from the present paper, the following conclusions can be taken.

1. *For α=0°, in the general system of reference $OX_0Y_0Z_0$ the reactions force and moments after axes directions OY_0 and OZ_0 vary with the angle θ_1, and in the local system of reference they remain constant.In contrast to the 4r asymmetrical spherical quadrilateral mechanism (from which derived the cardan joint mechanism) wherein all kinematic pairs appears an unbalanced axial force[6], the axial forces (after axe direction OX_0) from all kinematic pairs in the general system of reference have the zero value (they are balanced).Also as it was expected the reaction moment from the kinematic pair A is equal and opposite torque applied in D , and is evenly distributed at half from the value on the kinematic pairs 2, and 3.*
1. *For α≠0°, the reactions forces and moments aren't constant, both in the local and general system of reference. Their variation depends on the angle θ_1.*
2. *The values of the reaction forces and moments from the kinematic pairs are increasing as the angle between cardan shafts α increases.*

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