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THE CONTACT PRESSURE DISTRIBUTION ON THE TOOTHED CHAIN LINKS

Radu PAPUC[1], **Mihai – Tiberiu LATES**[2]
[1] Transilvania University of Brasov, radu.papuc@unitbv.ro
[2] Transilvania University of Brasov, latesmt@unitbv.ro

***Abstract*—**In many cases the surfaces through which various machine elements come in contact can be represented as cylinders which can be a good approximation of the real surfaces in contact. The paper presents the mathematical modeling of the pressure distribution exerted by the lubricant film on the contact length of the toothed chain link, using the unidimensional Reynolds equation.

***Keywords*—** toothed links, pressure, guide, Reynolds.

I. Introduction

ONE of the problems that attract the attention of the researchers in the tribological domain and of the machine construction is the hydrodynamic lubrication regime. The importance given to this friction regime is due to the many practical applications as well as the complexity of the theoretical and experimental problems which have to be solved [1].

Generally, in the machine construction, forces transmission is made by using relative big contact surfaces, a situation which leads to reduced pressures on the contact zone. This can ensure the existence of a continuous lubricant film which separates the friction surfaces, meaning the fluid lubrication is performed. Many years ago it was observed that in some heavy stressed friction joints and in which there is a weak geometrical conformation of the two elements surfaces (point and linear contacts), these behave as if they are lubricated in fluid regime (hydrodynamic). Practically, the hydrodynamic lubrication theory was not able to explain their functionality in reduced wear conditions [2]-[6].

II. The Contact Pressure Calculus

The tribological contact between the tensioning guide and the toothed chain is equivalent to the general case of two cylinders in relative motion with an interposer lubricant film. In the Fig. 1 there are presented the geometry and the functional parameters for the cylinder on cylinder fluid friction joint. For the studied case, it was considered the joint formed between two cylinders with radius R_1 and R_2 and relative speed U_1. It has to be mentioned that the cylinder with the radius R_1 represents the toothed chain link and is displaced with the speed U_1, while the cylinder with the radius R_2 represents the guide, which is fixed.

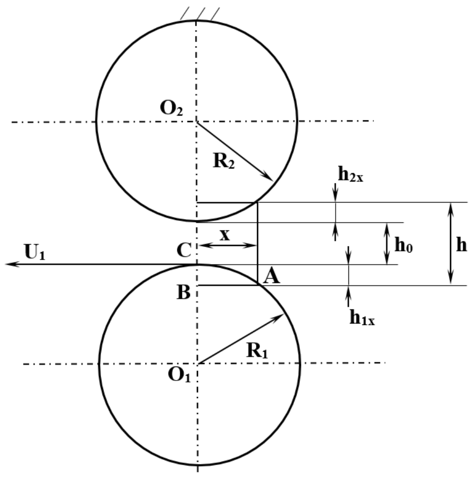

Fig.1 The geared chain - guide contact model

The displacement speed of the link towards the guide is computed considering the rotation n_1 and the division diameter D_{d1} of the chain gear as it follows:

$$U_1 = \frac{\pi}{60} \cdot n_1 \cdot D_{d1} \,. \qquad (1)$$

Considering the uniform distribution of the pressure on the lubricant film, the unidimensional Reynolds equation is [7], [8]

$$\frac{dp}{dx} = 12 \cdot \eta \cdot \bar{U} \cdot \frac{h - \bar{h}}{h^3} \qquad (2)$$

where:
1) η – is the oil viscosity;
2) h – is the lubricant film thickness;
3) $\bar{h}$ – is the lubricant film thickness at the maximum pressure point characterized by the position

$x = -\bar{x}((\frac{dp}{dx})_{x=-\bar{x}} = 0)$; the pressure will have a minimum p = 0 at the exit from the narrow space between the geared link and the guide, in the coordinate point $x = -\bar{x} \cdot ((\frac{dp}{dx})_{x=-\bar{x}} = 0)$, where $h = \bar{h}$ [9]-[11];

$\bar{U}$ is the medium speed of the lubrication film and is computed with the relation

$$\bar{\mathbf{U}} = \frac{1}{2} \cdot (\mathbf{U_1} + \mathbf{U_2}) = \frac{\mathbf{U_1}}{2}. \quad (3)$$

The lubricant film thickness *h*, in the coordinate *x* is given by the relation (Fig. 1)

$$\mathbf{h = h_0 + h_{1x} + h_{2x}} \quad (4)$$

where:

h_0 – is the minimum thickness of the lubricant film;

h_{1x}, h_{2x} – represents the thickness of the lubricant film considering the coordinate *x*:

$$\mathbf{h_{1x}} \approx \frac{\mathbf{x^2}}{\mathbf{2R_1}}, \quad (5)$$

$$\mathbf{h_{2x}} \approx \frac{\mathbf{x^2}}{\mathbf{2R_2}}. \quad (6)$$

The reduced curve for the two cylinders of radius R_1 and R_2 is given by the relation

$$\frac{\mathbf{1}}{\mathbf{R}} = \frac{\mathbf{1}}{\mathbf{R_1}} + \frac{\mathbf{1}}{\mathbf{R_2}}. \quad (7)$$

From the relations 5 – 8 it is obtained the thickness of the lubricant film

$$\mathbf{h = h_0} + \frac{\mathbf{x^2}}{\mathbf{2R}}. \quad (8)$$

By replacing the previously obtained relations in the general Reynolds equation, it is obtained the following form of the Reynolds equation:

$$\frac{\mathbf{dp}}{\mathbf{dx}} = \frac{\mathbf{6 \cdot \eta \cdot U_1}}{\mathbf{h_0^2}} \left[\frac{\mathbf{1}}{\left(\mathbf{1} + \frac{\mathbf{x^2}}{\mathbf{2Rh_0}}\right)^2} - \frac{\bar{\mathbf{h}}}{\mathbf{h_0}\left(\mathbf{1} + \frac{\mathbf{x^2}}{\mathbf{2Rh_0}}\right)^3} \right]. \quad (9)$$

Considering the Reynolds equation, the pressure p, which balances the external forces depends on the lubricant film thickness.

Determining the pressure distribution in the contact zone, the lubricant film form and the minimum thickness of this film, assumes solving the Reynolds differential equation, completed by the force expression transmitted by the joint considering the pressure distribution [12].

The following boundary conditions must be used in order to solve equation:

The pressure at exiting the narrow space, is equal to 0, $p(\bar{x}) = 0$; the pressure at entering the space $p\left(-\frac{p}{2}\right) = 0$;

The normal load on the contact is

$$F = b \int_{-\frac{1}{2}}^{\bar{x}} p dx, \quad (10)$$

where:

F – is the normal force on the links in the contact with the guide profile;

p_l – represents the chain step;

b – represents the contact width [13].

III. Numerical Results

The hypothesis where there are determined the parameters of the hydrodynamic film between the links and guide, according to the methodology proposed in this paper, are referring to:

1) The pressure of the lubricant film at the exit from the narrow area (the narrow area is that portion of the link and guide where the pressure is maximum) is equal to zero ($p(\bar{x}) = 0$);
2) The lubricant film pressure at the entry in the narrow zone between the geared link and the guide is defined by $p(\frac{p_1}{2}) = 0$.

The pressure distribution on the link guide contact length is obtained under the form presented in the Fig. 2.

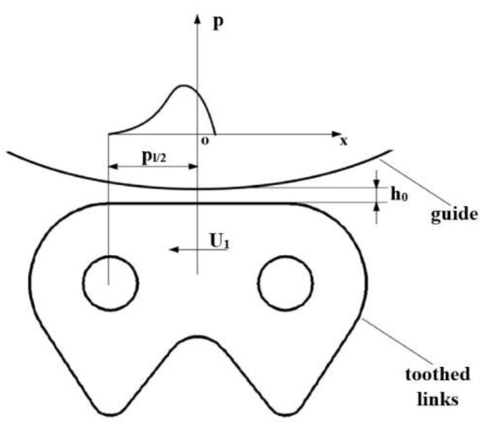

Fig. 2. The pressure distribution on the link guides contact length

Next there are presented the determined results of the hydrodynamic lubricant film parameters in successive points of the link guide contact, in the case where the guide radius is R = 0.2 (m) and the maximum normal pressure force on the guide is F_n = 175 (N).

The input data used to determine the pressure distribution in the contact between the tensioning guide and the geared chain are presented in table 1.

TABLE 1
THE INPUT DATA USED TO DETERMINE THE CONTACT PRESSURE

No.	Parameter	Symbol, value
1	Guide radius, m	R = 0.2
2	Driving wheel turation, rot/min	n_1 = 500...5000
3	Chain speed, m/s	U_1 = 1…10
4	Chain step, m	p = 0.0065
5	Teeth number of the chain wheel	z = 23
6	Link width, m	b_1 = 0.002
7	Viscosity, Pa·s	0.025
8	Chain link radius, m	$R_c = 5\cdot10^{-3}\ldots\infty$

Figure 3 shows the links position in contact with the guide profile. It has been traced with roman numerals the interior links and the sprocket guiding links in contact with the guide profile.

Also there were noted with Arabic numerals the exterior links position in contact with the guide profile.

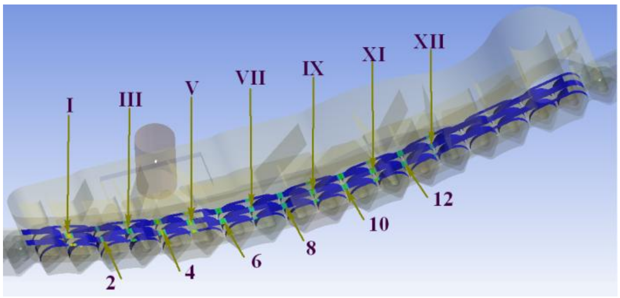

Fig. 3. The links position in contact with the guide profile

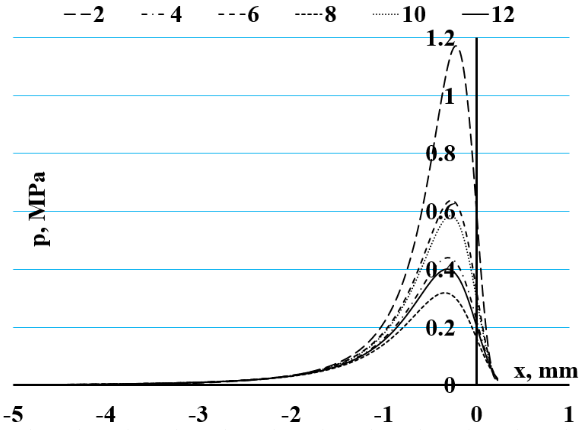

Fig. 4. The pressure distribution on the exterior links.

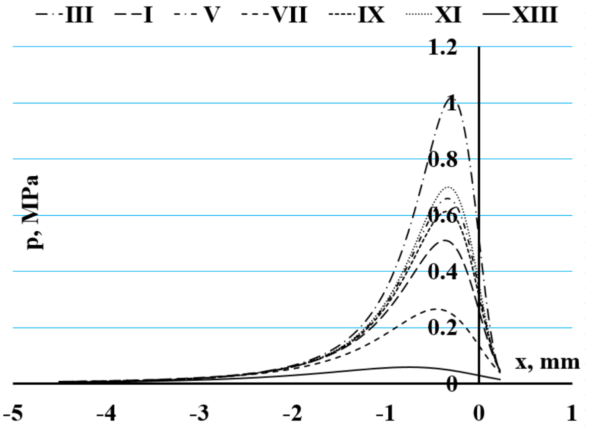

Fig. 5. The pressure distribution on the guiding sprockets and interior links.

Figures 4, 5 present the diagrams where can be seen the pressure distribution on the exterior links and the exterior and interior links.

It can be observed that the pressure distribution is bigger on the exterior link in contact with guide profile.

Figure 6, 7 present the variation of the minimum gap h_0 for the relative speed U = 5 (m/s).

This minimum gap h_0 is depending by normal force F_n = 175 (N), applied by the guide.

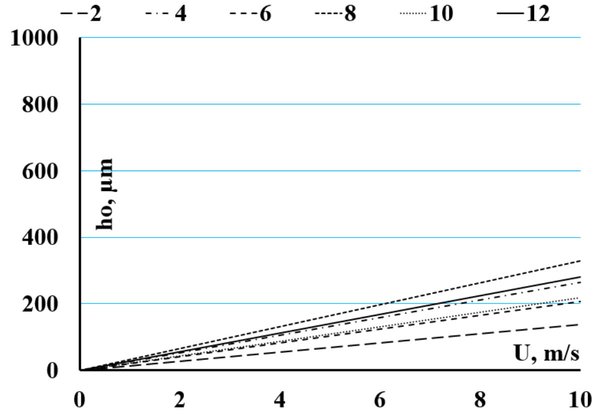

Fig. 6. Minimum gap h_0, depending on relative speed U = 5 (m/s) for exterior link.

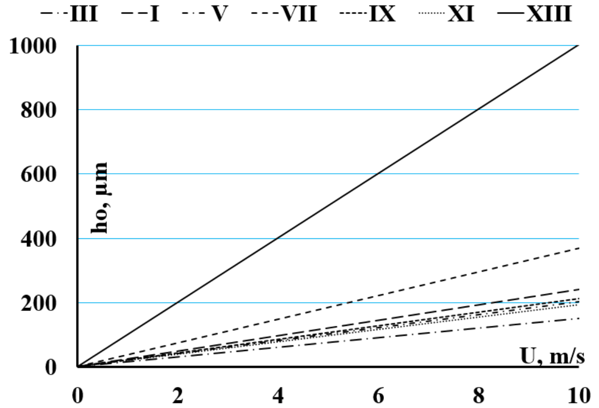

Fig. 7. Minimum gap h_0, depending on U = 5 (m/s) for guiding sprockets link and interior link.

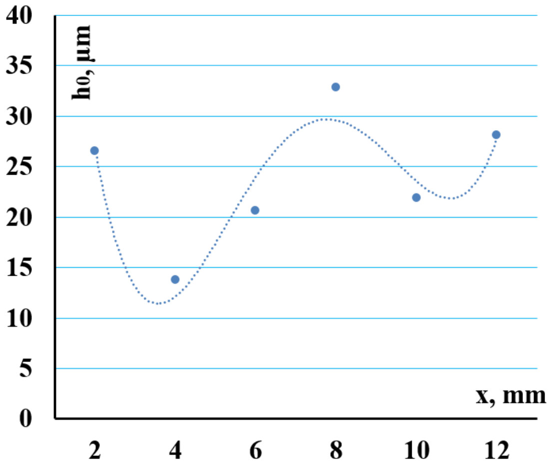

Fig. 8. The minimum thickness of the lubricant film on the guiding link and interior link for relative speed U = 5 (m/s).

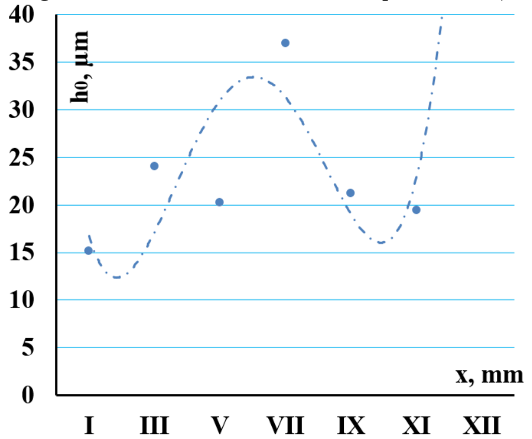

Fig. 9. The minimum thickness of the lubricant film on the exterior link for relative speed U = 5 (m/s).

The variation of the gap h_0 with the links position is presented in Fig. 8 (for the exterior links) and in Fig. 9 (for the guiding and interior links).

The analyze of the lubricant film parameters resulted after the solving of Reynolds equation, gives information in order to understand better the transition between the boundary and the hydrodynamic lubrication, in the case of the contact between the guide and chain

IV. CONCLUSION

From the results determined through the tribological method of the contact between the tensioning guide and the geared chain, and the lubricant film distribution, the following conclusions are drawn:

1) *when passing the chain link over the tensioning guide it is crossed a rising and decreasing pressure process; for a chain speed of 5 (m/s), there were resulted in pressure peaks of maximum 2 (MPa);*
2) *the contribution of the viscosity to the external forces equilibrium is apparently reduced, but by analyzing the pressure equations results that the pressure is entirely determined by viscosity, so, the lubricant film forming process is actually determined by the lubricant viscosity;*
3) *the pressure peaks also determine the minimum values of the lubricant film thickness, for a relative chain speed of 5 m/s, it will be obtained minimum values of the lubricant film 10-15 (μm) for the guide radius of 0.2 (m), these values decreasing proportionally with the speed decreasing;*
4) *the contact pressures are slightly bigger on the exterior links 6;*
5) *the minimum lubricant film thickness was obtained on the exterior links 4;*
6) *increasing the guide radius determine a pulsatory variation tendency of the pressure in the crossing process of the link over the guide, with two or even three pressure peaks.*

The calculated thick of the lubricant film indicates that the guide-link contact, according to the functioning conditions, is a mixed friction and a fluid friction contact. Fluid friction is obtained in the case of big radiuses of the chain guide, small tensioning forces and high rotational speeds. For small rotational speeds and big tensioning forces the limit and the mixed friction should be accepted.

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