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CALCULATION ALGORITHM FOR DETERMINING THE EFFORTS FROM CARDAN JOINT MECHANISM WITHOUT TECHNICAL DEVIATIONS

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Abstract—The normal cardan joint with multiple technical applications is a particular case of the RRRR mechanism in which the axes of cardan cross are perpendicular. Being of the third family the mechanism with one cardan joint is multiple statically indeterminate.To calculate the reactions from kinematic pairs is used elastic linear calculation.

In this paper is established the calculation algorithm for determining the efforts of the ideal (without technical deviations) cardan joint mechanism and using the numerical calculation methods allows determining the reactions from the kinematic pairs.

The results of the numerical solving of this problem will be presented under the form of a diagrams and will be commented.

Keywords—cardan joint, cardan transmission, elastic calculation, quadrilateral mechanism.

I. INTRODUCTION

UNDER the influence of external loads, the elements of the mechanism are deformed. The elastic displacements are small and defined by the vectors of small rotation, which act similarly to the angular velocity and small displacements, defined by displacement vectors. Expressing them in plücker coordinates and applied the relative displacements method make possible the determination of efforts and the reactions from the kinematic pairs of the mechanism, multiple statically indeterminate [1]-[4].

II. GENERAL ASPECTS

The mechanically speaking (static) , the 4R mechanism has unknown the reactions from the kinematic pairs A, 2, 3, 5, 6, 8 and also the moment from the joint A (see Fig. 1.), in total 31 unknowns and 18 equations, result in 13 times statically undetermined.

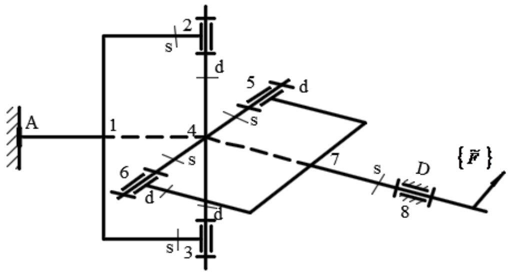

Fig. 1. The 4r Symmetrical Spherical Quadrilateral Mechanism.

For determining these components a linear elastic calculation is used. In the following, a mathematical model with being elaborated for the linear elastic calculation of reactions, a model that has as basis the method of relative displacements, presented in paper [5], with the notation in plücker coordinates.

In the elastic calculation the joint from A is blocked (see Fig. 1.), the thing that explains the apparition as unknown of the axial moment at that point.

The pivotal points 1, 4, 7, have the displacements $\{\Delta_1\}$, $\{\Delta_4\}$, $\{\Delta_7\}$, and the pivotal points with the cinematic couples 2, 3, 5, 6 have left-right displacements with $\{\Delta_2^s\}$, $\{\Delta_2^d\}$, $\{\Delta_3^s\}$, $\{\Delta_3^d\}$, $\{\Delta_5^s\}$, $\{\Delta_5^d\}$, $\{\Delta_6^s\}$, $\{\Delta_6^d\}$. Pivotal point 8 has the displacement to the left $\{\Delta_8^s\}$ and to the right $\{\Delta_8^d\}=\{0\}$.So the relations are written under the form:

$$\begin{cases}\{\Delta_2^d\}=\{\Delta_2^s\}+\xi_2\{U_2\}\\ \{\Delta_3^d\}=\{\Delta_3^s\}+\xi_3\{U_3\}\\ \{\Delta_5^d\}=\{\Delta_5^s\}+\xi_5\{U_5\}\\ \{\Delta_6^d\}=\{\Delta_6^s\}+\xi_6\{U_6\}\\ \{0\}=\{\Delta_8^s\}+\xi_7\{U_7\}.\end{cases} \tag{1}$$

where $\{U_i\}$, $i=2,3,5,6,8$,are the column matrixes of the plücker coordinates attached to the kinematic pairs $[5],[6]$.

From the pivotal points balance results

$$\begin{cases} [K_{12}]\left(\{\Delta_1\}-\{\Delta_2^s\}\right)+[K_{1A}]\left(\{\Delta_1\}-\{\Delta_A^0\}\right)+ \\ \qquad +[K_{13}]\left(\{\Delta_1\}-\{\Delta_3^s\}\right)=\{0\} \\ [K_{21}]\left(\{\Delta_2^s\}-\{\Delta_1\}\right)+[K_{24}]\left(\{\Delta_2^d\}-\{\Delta_4\}\right)=\{0\} \\ [K_{31}]\left(\{\Delta_3^s\}-\{\Delta_1\}\right)+[K_{34}]\left(\{\Delta_3^d\}-\{\Delta_4\}\right)=\{0\} \\ [K_{42}]\left(\{\Delta_4\}-\{\Delta_2^d\}\right)+[K_{43}]\left(\{\Delta_4\}-\{\Delta_3^d\}\right)+ \\ +[K_{45}]\left(\{\Delta_4\}-\{\Delta_5^s\}\right)+[K_{46}]\left(\{\Delta_4\}-\{\Delta_6^s\}\right)=\{0\} \\ [K_{54}]\left(\{\Delta_5^s\}-\{\Delta_4\}\right)+[K_{57}]\left(\{\Delta_5^d\}-\{\Delta_7\}\right)=\{0\} \\ [K_{64}]\left(\{\Delta_6^s\}-\{\Delta_4\}\right)+[K_{67}]\left(\{\Delta_6^d\}-\{\Delta_7\}\right)=\{0\} \\ [K_{75}]\left(\{\Delta_7\}-\{\Delta_5^d\}\right)+[K_{76}]\left(\{\Delta_7\}-\{\Delta_6^d\}\right)+ \\ \qquad +[K_{78}]\left(\{\Delta_7\}-\{\Delta_8^s\}\right)=\{0\}. \end{cases} \tag{2}$$

$[Kij]=[Kji]$, where $[Kij]$, is the rigidity matrix of the ij element. Are noted

$$\begin{aligned} &[\mathbf{K}_{11}]=[\mathbf{K}_{1A}]+[\mathbf{K}_{12}]+[\mathbf{K}_{13}] \\ &[\mathbf{K}_{22}]=[\mathbf{K}_{21}]+[\mathbf{K}_{24}] \\ &[\mathbf{K}_{33}]=[\mathbf{K}_{31}]+[\mathbf{K}_{34}]; \\ &[\mathbf{K}_{44}]=[\mathbf{K}_{42}]+[\mathbf{K}_{43}]+[\mathbf{K}_{45}]+[\mathbf{K}_{46}] \\ &[\mathbf{K}_{55}]=[\mathbf{K}_{54}]+[\mathbf{K}_{57}];\ [\mathbf{K}_{66}]=[\mathbf{K}_{64}]+[\mathbf{K}_{67}] \\ &[\mathbf{K}_{77}]=[\mathbf{K}_{75}]+[\mathbf{K}_{76}]+[\mathbf{K}_{78}]. \end{aligned} \tag{3}$$

and taking into account the relations (1) it results

$$\begin{cases} [\mathbf{K}_{11}]\{\Delta_1\}-[\mathbf{K}_{12}]\{\Delta_2^s\}-[\mathbf{K}_{13}]\{\Delta_3^s\}=\{0\} \\ [\mathbf{K}_{21}]\{\Delta_1\}+[\mathbf{K}_{22}]\{\Delta_2^s\}-[\mathbf{K}_{24}]\{\Delta_4\}+ \\ \qquad +\xi_2[\mathbf{K}_{24}]\{\mathbf{U}_2\}=\{0\} \\ -[\mathbf{K}_{31}]\{\Delta_1\}+[\mathbf{K}_{33}]\{\Delta_3^s\}-[\mathbf{K}_{34}]\{\Delta_4\}+ \\ \qquad +\xi_3[\mathbf{K}_{34}]\{\mathbf{U}_3\}=\{0\} \\ [\mathbf{K}_{31}]\{\Delta_1\}+[\mathbf{K}_{33}]\{\Delta_3^s\}-[\mathbf{K}_{34}]\{\Delta_4\}+ \\ \qquad +\xi_3[\mathbf{K}_{34}]\{\mathbf{U}_3\}=\{0\} \\ -[\mathbf{K}_{54}]\{\Delta_4\}-[\mathbf{K}_{55}]\{\Delta_5^s\}-[\mathbf{K}_{57}]\{\Delta_7\}+ \\ \qquad +\xi_5[\mathbf{K}_{57}]\{\mathbf{U}_5\}=\{0\} \\ -[\mathbf{K}_{64}]\{\Delta_4\}+[\mathbf{K}_{66}]\{\Delta_6^s\}-[\mathbf{K}_{67}]\{\Delta_7\}+ \\ \qquad +\xi_6[\mathbf{K}_{67}]\{\mathbf{U}_6\}=\{0\} \\ [\mathbf{K}_{75}]\{\Delta_5^s\}-[\mathbf{K}_{76}]\{\Delta_6^s\}+[\mathbf{K}_{77}]\{\Delta_7\}-\xi_5[\mathbf{K}_{75}]\{\mathbf{U}_5\}- \\ \qquad -\xi_6[\mathbf{K}_{76}]\{\mathbf{U}_6\}+\xi_8[\mathbf{K}_{78}]\{\mathbf{U}_8\}=\{0\}. \end{cases} \tag{4}$$

The notations are made as

$$\{\Delta\}=\left[\{\Delta_1\}^{\mathbf{T}},\{\Delta_2^s\}^{\mathbf{T}},\{\Delta_3^s\}^{\mathbf{T}},\{\Delta_4\}^{\mathbf{T}},\{\Delta_5^s\}^{\mathbf{T}},\{\Delta_6^s\}^{\mathbf{T}},\{\Delta_7\}^{\mathbf{T}}\right]^{\mathbf{T}}$$

$$\{\xi\}=[\xi_2,\xi_3,\xi_5,\xi_6,\xi_8]^{\mathbf{T}} \tag{5}$$

$$[\mathbf{V}_2]=[1,0,0,0,0];\ [\mathbf{V}_3]=[0,1,0,0,0];\ [\mathbf{V}_5]=[0,0,1,0,0]$$

$$[\mathbf{V}_6]=[0,0,0,1,0];\ [\mathbf{V}_8]=[0,0,0,0,1].$$

ξ_i , $[\xi]$ being the elastic rotations respectively the elastic rotations matrix from the kinematic pairs [5],[6] and $[V_i]$ the matrix that verifies the relation

$$\zeta_{\mathbf{i}}=[\mathbf{V}_{\mathbf{i}}]\{\xi\}. \tag{6}$$

and $\zeta_2[K_{24}]\{U_2\}=[K_{24}]\{U_2\}[V_2]\{\xi\}$, and the analogue.

Also, the notations are made

$$[\mathbf{K}_1]=\begin{bmatrix} [\mathbf{K}_{11}] & -[\mathbf{K}_{12}] & -[\mathbf{K}_{13}] & [0] & [0] & [0] & [0] \\ -[\mathbf{K}_{21}] & [\mathbf{K}_{22}] & [0] & -[\mathbf{K}_{24}] & [0] & [0] & [0] \\ -[\mathbf{K}_{31}] & [0] & [\mathbf{K}_{33}] & [\mathbf{K}_{34}] & [0] & [0] & [0] \\ [0] & -[\mathbf{K}_{42}] & -[\mathbf{K}_{43}] & [\mathbf{K}_{44}] & -[\mathbf{K}_{45}] & -[\mathbf{K}_{46}] & [0] \\ [0] & [0] & -[\mathbf{K}_{54}] & [0] & [\mathbf{K}_{55}] & [0] & -[\mathbf{K}_{57}] \\ [0] & [0] & -[\mathbf{K}_{64}] & [0] & [0] & [\mathbf{K}_{66}] & -[\mathbf{K}_{67}] \\ [0] & [0] & [0] & [0] & -[\mathbf{K}_{75}] & -[\mathbf{K}_{76}] & [\mathbf{K}_{77}] \end{bmatrix}. \tag{7}$$

$$[\mathbf{K}_2]=\begin{bmatrix} [0] \\ [\mathbf{K}_{24}]\{\mathbf{U}_2\}[\mathbf{V}_2] \\ [\mathbf{K}_{34}]\{\mathbf{U}_3\}[\mathbf{V}_3] \\ -[\mathbf{K}_{24}]\{\mathbf{U}_2\}[\mathbf{V}_2]-[\mathbf{K}_{34}]\{\mathbf{U}_3\}[\mathbf{V}_3] \\ [\mathbf{K}_{57}]\{\mathbf{U}_5\}[\mathbf{V}_5] \\ [\mathbf{K}_{67}]\{\mathbf{U}_6\}[\mathbf{V}_6] \\ -[\mathbf{K}_{57}]\{\mathbf{U}_5\}[\mathbf{V}_5]-[\mathbf{K}_{67}]\{\mathbf{U}_6\}[\mathbf{V}_6]+ \\ +[\mathbf{K}_{78}]\{\mathbf{U}_8\}[\mathbf{V}_8] \end{bmatrix}. \tag{8}$$

and then the equations (4) are combined into the equation

$$[\mathbf{K}_1]\{\Delta\}+[\mathbf{K}_2]\{\xi\}=\{\mathbf{0}\}. \tag{9}$$

equivalent with 42 scalar equations.Isolating the left side of the pair 2, (see Fig. 2.) results that

$$\{\mathbf{R}_2\}=\{\mathbf{E}_2^s\}=[\mathbf{K}_{21}](\{\Delta_2^s\}-\{\Delta_1\}). \tag{10}$$

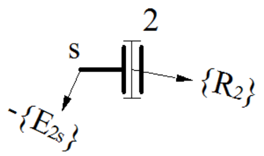

Fig. 2. The isolation of couple 2.

$[R_i]$, $\{E_i^s\}$ being the column matrixes of the plücker coordinates of the reactions from the kinematic pairs [5], [6] respectively the column matrixes of the efforts from the bar from the left of the kinematic pairs. And the analogue

$$\begin{cases}\{\mathbf{R}_3\}=\{\mathbf{E}_3^s\}=[\mathbf{K}_{31}](\{\Delta_3^s\}-\{\Delta_1\})\\ \{\mathbf{R}_5\}=\{\mathbf{E}_5^s\}=[\mathbf{K}_{54}](\{\Delta_5^s\}-\{\Delta_4\})\\ \{\mathbf{R}_6\}=\{\mathbf{E}_6^s\}=[\mathbf{K}_{64}](\{\Delta_6^s\}-\{\Delta_4\})\\ \{\mathbf{R}_8\}=\{\mathbf{E}_8^s\}-\{\mathbf{F}\}=[\mathbf{K}_{87}](\{\Delta_8^s\}-\{\Delta_7\})-\{\mathbf{F}\}.\end{cases} \tag{11}$$

$[F]$,being the column matrixes of the plücker coordinates of the external load.

As [5], $\{\tilde{U}_i\}^T\{R_i\}=0$ and $\{\Delta_8^s\}=-\xi_8\{U_8\}$ the equations are obtained

$$\begin{cases}\{\tilde{\mathbf{U}}_2\}^T[\mathbf{K}_{21}]\{\Delta_2^s\}-\{\tilde{\mathbf{U}}_2\}^T[\mathbf{K}_{21}]\{\Delta_1\}=\mathbf{0}\\ \{\tilde{\mathbf{U}}_3\}^T[\mathbf{K}_{31}]\{\Delta_3^s\}-\{\tilde{\mathbf{U}}_3\}^T[\mathbf{K}_{31}]\{\Delta_1\}=\mathbf{0}\\ \{\tilde{\mathbf{U}}_5\}^T[\mathbf{K}_{54}]\{\Delta_5^s\}-\{\tilde{\mathbf{U}}_5\}^T[\mathbf{K}_{54}]\{\Delta_4\}=\mathbf{0}\\ \{\tilde{\mathbf{U}}_6\}^T[\mathbf{K}_{64}]\{\Delta_6^s\}-\{\tilde{\mathbf{U}}_6\}^T[\mathbf{K}_{64}]\{\Delta_4\}=\mathbf{0}\\ \{\tilde{\mathbf{U}}_8\}^T[\mathbf{K}_{87}]\{\Delta_7\}+\xi_8\{\tilde{\mathbf{U}}_8\}^T[\mathbf{K}_{87}]\{\mathbf{U}_8\}=\{\tilde{\mathbf{U}}_8\}^T[\mathbf{F}].\end{cases} \tag{12}$$

With the notations

$$[\mathbf{K}_3]=\begin{bmatrix}\{\tilde{\mathbf{U}}_2\}^T[\mathbf{K}_{21}] & -\{\tilde{\mathbf{U}}_2\}^T[\mathbf{K}_{21}] & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \{\tilde{\mathbf{U}}_3\}^T[\mathbf{K}_{31}] & \mathbf{0} & -\{\tilde{\mathbf{U}}_3\}^T[\mathbf{K}_{31}] & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \{\tilde{\mathbf{U}}_5\}^T[\mathbf{K}_{54}] & -\{\tilde{\mathbf{U}}_5\}^T[\mathbf{K}_{54}] & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \{\tilde{\mathbf{U}}_6\}^T[\mathbf{K}_{64}] & \mathbf{0} & -\{\tilde{\mathbf{U}}_6\}^T[\mathbf{K}_{64}] & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \{\tilde{\mathbf{U}}_8\}^T[\mathbf{K}_{78}]\end{bmatrix}. \tag{13}$$

$$[\mathbf{K}_4]=\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \{\tilde{\mathbf{U}}_8\}^T[\mathbf{K}_{78}]\{\mathbf{U}_8\}\end{bmatrix};\{\tilde{\mathbf{F}}\}=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \{\tilde{\mathbf{U}}_8\}^T[\mathbf{F}]\end{bmatrix}. \tag{14}$$

the equations (11) are combined in the matrix equation

$$[\mathbf{K}_3]\{\Delta\}+[\mathbf{K}_4]\{\xi\}=\{\tilde{\mathbf{F}}\}. \tag{15}$$

equivalent with 5 scalar equations.

The equations (7), (15) can be narrowed with the notations

$$[\mathbf{K}]=\begin{bmatrix}[\mathbf{K}_1]_1 & [\mathbf{K}_2]\\ [\mathbf{K}_3] & [\mathbf{K}_4]\end{bmatrix}. \tag{16}$$

in the equation

$$[\mathbf{K}]\begin{bmatrix}\{\Delta\}\\ \{\xi\}\end{bmatrix}=\begin{bmatrix}\{\mathbf{0}\}\\ \{\tilde{\mathbf{F}}\}\end{bmatrix}. \tag{17}$$

equivalent with 47 equations with 47 unknowns from which results $\{\Delta\}$ and $\{\xi\}$, and the reactions and efforts are calculated with the relations (10), (11) in which $\{\Delta_8^s\}=-\xi_8\{U_8\}$.

III. CALCULATION ALGORITHM

1) The indexation of bars is done from $i=1,2,\ldots,14$ and the lengths are noted with $l_i, i=1,2,\ldots,14$, with $l_2=l_4$; $l_3=l_5$; $l_6=l_7$; $l_8=l_9$; $l_{10}=l_{12}$; $l_{11}=l_{13}$.

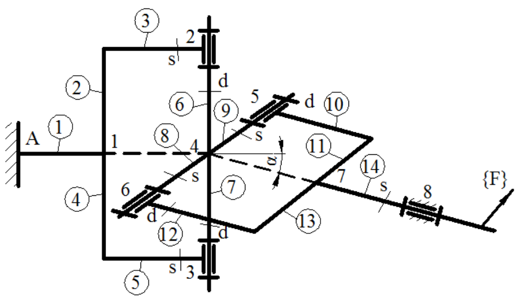

Fig. 3. Indexing bars.

2) The inertia moments is calculated:

$$\mathbf{I}_{iy}=\mathbf{I}_{iz}=\frac{\pi \mathbf{d}_i^4}{64};\mathbf{I}_{ix}=\mathbf{I}_{iy}+\mathbf{I}_{iz};A_i=\frac{\pi \mathbf{d}_i^2}{4}. \tag{18}$$

3) The rigidity, flexibility matrixes $[k_i]$, $[h_i]$ are calculated in the local reference system:

$$[\mathbf{k_i}]=\begin{bmatrix} 0 & 0 & 0 & \frac{E_iA_i}{l_i} & 0 & 0 \\ 0 & 0 & \frac{6E_iI_{iz}}{l_i^2} & 0 & \frac{12E_iI_{iz}}{l_i^3} & 0 \\ 0 & -\frac{6E_iI_{iy}}{l_i^2} & 0 & 0 & 0 & \frac{12E_iI_{iy}}{l_i^3} \\ \frac{G_iI_{ix}}{l_i} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{4E_iI_{iy}}{l_i} & 0 & 0 & 0 & -\frac{6E_iI_{iy}}{l_i^2} \\ 0 & 0 & \frac{4E_iI_{iz}}{l_i} & 0 & \frac{6E_iI_{iz}}{l_i^2} & 0 \end{bmatrix}. \quad (19)$$

$$[\mathbf{h_i}]=\begin{bmatrix} 0 & 0 & 0 & \frac{l_i}{G_iI_{ix}} & 0 & 0 \\ 0 & 0 & \frac{l_i^2}{2E_iI_{iy}} & 0 & \frac{l_i}{E_iI_{iy}} & 0 \\ 0 & -\frac{l_i^2}{2E_iI_{iz}} & 0 & 0 & 0 & \frac{l_i}{E_iI_{iz}} \\ \frac{l_i}{E_iA_i} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{l_i^3}{3E_iI_{iz}} & 0 & 0 & 0 & -\frac{l_i^2}{2E_iI_{iz}} \\ 0 & 0 & \frac{l_i^3}{3E_iI_{iy}} & 0 & \frac{l_i^2}{2E_iI_{iy}} & 0 \end{bmatrix}. \quad (20)$$

where, E_i, G_i are the elasticity modulus.

4) The rotation, translation matrixes $[R_i]$; $[G_i]$ are calculated, with the relations from TABLE 4.3.1, [7] and then the product $[G_i]\cdot[R_i]$.

5) The position matrixes are calculated [5], [6] :

$$[\mathbf{T_i}]=\begin{bmatrix} [\mathbf{R_i}] & [\mathbf{0}] \\ [\mathbf{G_i}][\mathbf{R_i}] & [\mathbf{R_i}] \end{bmatrix}$$

$$[\mathbf{T_i}]^{-1}=\begin{bmatrix} [\mathbf{R_i}]^T & [\mathbf{0}] \\ [\mathbf{R_i}]^T[\mathbf{G_i}]^T & [\mathbf{R_i}]^T \end{bmatrix}. \quad (21)$$

$[R_i]$, $[G_i]$, $[T_i]$, are the rotation, translation, position matrixes of the local systems of reference $O_ix_iy_iz_i$ in relation to the mobile systems of reference $OX'Y_0Z_0'$, OX_0Y^*Z , $OX'YZ_0^{\bullet}$, defined in the paper [6].

6) The flexibility, rigidity matrixes $[H_i^*]$, $[K_i^*]$ in relation to the mobile reference systems $OX'Y_0Z_0'$, OX_0Y^*Z , $OX'YZ_0^{\bullet}$ are calculated [5], [6] :

$$[\mathbf{H_i^*}]=[\mathbf{T_i}][\mathbf{h_i}][\mathbf{T_i}]^{-1} \ ; \ [\mathbf{K_i^*}]=[\mathbf{T_i}][\mathbf{k_i}][\mathbf{T_i}]^{-1}. \quad (22)$$

7) The matrixes $[H_{23}^*]$, $[H_{45}^*]$, $[H_{10,11}^*]$, $[H_{12,13}^*]$ are calculated:

$$[\mathbf{H_{23}^*}]=[\mathbf{H_2^*}]+[\mathbf{H_3^*}]\ ;\ [\mathbf{H_{45}^*}]=[\mathbf{H_4^*}]+[\mathbf{H_5^*}]$$
$$[\mathbf{H_{10,11}^*}]=[\mathbf{H_{10}^*}]+[\mathbf{H_{11}^*}]\ ;\ [\mathbf{H_{12,13}^*}]=[\mathbf{H_{12}^*}]+[\mathbf{H_{13}^*}] \quad (23)$$

8) It is identified:

$$[\mathbf{K_{1A}^*}]=[\mathbf{K_{11}^*}]\ ;\ [\mathbf{K_{12}^*}]=[\mathbf{H_{23}^*}]^{-1};\ [\mathbf{K_{13}^*}]=[\mathbf{H_{45}^*}]^{-1}$$
$$[\mathbf{K_{24}^*}]=[\mathbf{K_6^*}]\ ;\ [\mathbf{K_{34}^*}]=[\mathbf{K_7^*}]\ ;\ [\mathbf{K_{45}^*}]=[\mathbf{K_9^*}]$$
$$[\mathbf{K_{46}^*}]=[\mathbf{K_8^*}]\ ;\ [\mathbf{K_{57}^*}]=[\mathbf{H_{10,11}^*}]^{-1};\ [\mathbf{K_{67}^*}]=[\mathbf{H_{12,13}^*}]^{-1} \quad (24)$$
$$[\mathbf{K_{67}^*}]=[\mathbf{H_{12,13}^*}]^{-1};\ [\mathbf{K_{78}^*}]=[\mathbf{K_{14}^*}].$$

9) θ_2 is calculated with the formulas:

$$\theta_2=\begin{cases} \mathrm{arctg}\left(\frac{1}{c\alpha}\mathrm{tg}\theta_1\right); 0\le\theta_1<\frac{\pi}{2} \\ \frac{\pi}{2}; \theta_1=\frac{\pi}{2} \\ \pi+\mathrm{arctg}\left(\frac{1}{c\alpha}\mathrm{tg}\theta_1\right); \frac{\pi}{2}\le\theta_1<\frac{3\pi}{2} \\ \frac{3\pi}{2}; \theta_1=\frac{3\pi}{2} \\ 2\pi+\mathrm{arctg}\left(\frac{1}{c\alpha}\mathrm{tg}\theta_1\right); \frac{3\pi}{2}<\theta_1\le 2\pi. \end{cases} \quad (25)$$

where θ_1 is the rotation angle of the input driving, θ_2 the rotation angle of the output driven, α the angle between the cardan shafts and by c noted the trigonometric function *cos* .

10) $[R_{AB}]$, $[R_{BC}]$, $[R_{CD}]$, $[T_{AB}]$, $[T_{BC}]$, $[T_{CD}]$ the rotation, position matrixes of the mobile systems of reference $OX'Y_0Z_0'$, OX_0Y^*Z , $OX'YZ_0^{\bullet}$, which contains the input driving, the cardan cross and the output driven in relation to the system $OX_0Y_0Z_0$ are calculated with the relations [5], [6]:

$$[\mathbf{T_{AB}}]=\begin{bmatrix} [\mathbf{R_{AB}}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{R_{AB}}] \end{bmatrix}$$
$$[\mathbf{T_{BC}}]=\begin{bmatrix} [\mathbf{R_{BC}}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{R_{BC}}] \end{bmatrix};\ [\mathbf{T_{CD}}]=\begin{bmatrix} [\mathbf{R_{CD}}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{R_{CD}}] \end{bmatrix}. \quad (26)$$

$$[\mathbf{R_{AB}}]=\begin{bmatrix}1 & 0 & 0\\ 0 & c\theta_1 & -s\theta_1\\ 0 & s\theta_1 & c\theta_1\end{bmatrix},[\mathbf{R_{CD}}]=\begin{bmatrix}c\alpha & s\alpha s\theta_2 & s\alpha c\theta_2\\ 0 & c\theta_2 & -s\theta_2\\ -s\alpha & c\alpha s\theta_2 & c\alpha c\theta_2\end{bmatrix} \quad (27)$$

$$[\mathbf{R_{BC}}]=\begin{bmatrix}c\theta_1c\theta_2+s\theta_1s\theta_2c\alpha & s\alpha s\theta_2 & 0\\ -c\theta_1s\theta_2s\alpha & c\theta_2 & -s\theta_1\\ -s\theta_1s\theta_2s\alpha & c\alpha s\theta_2 & c\theta_1\end{bmatrix}.$$

whereby *s* noted the trigonometric functions *sin*.

11) The rigidity matrixes in relation to the general system of reference $OX_0Y_0Z_0$ are calculated:

$[K_{1A}],[K_{12}],[K_{13}]$ for the input driving with the formula

$$[\mathbf{K}]=[\mathbf{T_{AB}}][\mathbf{K}^*][\mathbf{T_{AB}}]^{-1}. \quad (28)$$

$[K_{24}],[K_{34}],[K_{45}],[K_{46}]$ for the cardan cross with formula type

$$[\mathbf{K}]=[\mathbf{T_{BC}}][\mathbf{K}^*][\mathbf{T_{BC}}]^{-1}. \quad (29)$$

$[K_{57}],[K_{67}],[K_{78}]$ for the output driven with formula type

$$[\mathbf{K}]=[\mathbf{T_{CD}}][\mathbf{K}^*][\mathbf{T_{CD}}]^{-1}. \quad (30)$$

12) Are calculated:

$$\{\mathbf{U_2}\}=\{\mathbf{U_3}\}=[0,-s\theta_1,c\theta_1,0,0,0]^T$$
$$\{\mathbf{U_5}\}=\{\mathbf{U_6}\}=[s\theta_2s\alpha,c\theta_2,s\theta_2c\alpha,0,0,0]^T \quad (31)$$
$$\{\mathbf{U_8}\}=[c\alpha,0,-s\alpha,0,0,0]^T.$$

$$\{\tilde{\mathbf{U}}_2\}=\{\tilde{\mathbf{U}}_3\}=[0,0,0,0,-s\theta_1,c\theta_1]^T$$
$$\{\tilde{\mathbf{U}}_5\}=\{\tilde{\mathbf{U}}_6\}=[0,0,0,s\theta_2s\alpha,c\theta_2,s\theta_2c\alpha]^T \quad (32)$$
$$\{\tilde{\mathbf{U}}_8\}=[0,0,0,c\alpha,0,-s\alpha]^T.$$

$$\{\mathbf{F}\}=\tilde{\mathbf{M}}[0,0,0,c\alpha,0,-s\alpha]^T \;;\; \{\tilde{\mathbf{U}}_8\}^T\{\mathbf{F}\}=\tilde{\mathbf{M}}. \quad (33)$$

13) Are calculated the matrixes $[K_1],[K_2],[K_3],[K_4],\{\tilde{F}\},[K]$ with the formulas (7), (8), (13), (14), (16).

14) It is solved the matrix equation (17).

15) The reaction from A is calculated with the relation

$$\{\mathbf{R_A}\}=-[\mathbf{K_{1A}}]\{\Delta_1\}. \quad (34)$$

and expressed in local coordinates.

16) The reactions are calculated $\{R_2\},\{R_3\},\{R_5\},\{R_6\},\{R_8\}$ with the relations (10), (11).

17) The reactions are expressed under $\{R_2\},\{R_3\},\{R_5\},\{R_6\},\{R_8\}$ in the local system coordinates.

18) The graphs ξ_i, $i=2,3,5,6,8$ and R_{ix},M_{ix}, $i=A,2,3,5,6,8$ are made taking into account θ_1.

IV. NUMERICAL APPLICATION

It is considered the cardan joint mechanism from Fig. 3. The mechanism's elements have the following geometrical and mechanical characteristics: the lengths $l_i=l=0{,}06(m)$, the diameters $d_i=d=0{,}02(m)$, the sections A_i, the elasticity modulus $E=2{,}1\cdot10^{11}(N/m^2)$, $G=8{,}1\cdot10^{10}(N/m^2)$, and the main central inertial moments I_{iy},I_{iz}, $I_{ix}=I_{iy}+I_{iz}$, $i=1,2,\ldots,14$. The mechanism is driven by torque $\tilde{M}=1(N\cdot m)$. On the basis the mathematical model which will be presented in the next paper and the algorithm presented in this paper has been realized in Excel a calculation program of the reactions from the kinematic pairs. As a result of numerical simulations were obtained the following results.

In the case where the angle between the cardan shafts is $\alpha=0^\circ$, the reaction forces and moments $R^0_{2X},R^0_{2Y},R^0_{2Z}$, $M^0_{2X},M^0_{2Y},M^0_{2Z}$ from the kinematic pair 2, in the general system of reference varies as shown in Fig. 4. and Fig. 5. and in the local system of reference the reaction forces $R^0_{2X},R^0_{2Y},R^0_{2Z}$ varies as shown in Fig. 6.

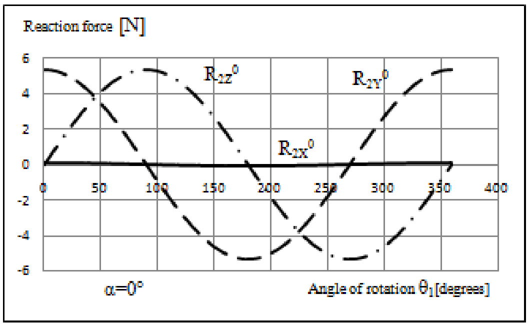

Fig. 4. The variation diagrams of reaction forces in the general reference system in the kinematic pair 2, for *α=0°*.

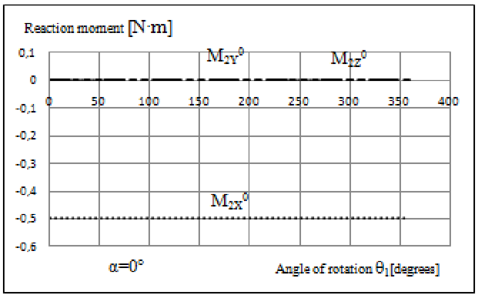

Fig. 5. The variation diagrams of reaction moments in the general reference system in the kinematic pair 2, for *α=0°*.

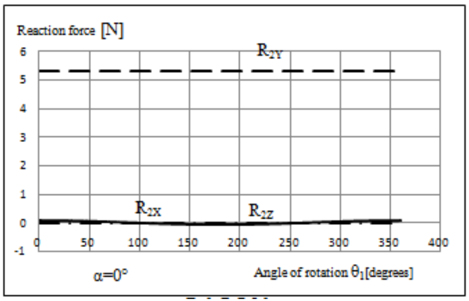

Fig. 6. The variation diagrams of reaction forces in the local reference system in the kinematic pair 2, for *α=0°*.

From the variation diagrams it is found that the reaction forces and moments are constant in the local systems of reference for all kinematic pairs and in the general system of reference remain constant.

The variation of the components $R_{2X}^0, R_{2Y}^0, R_{2Z}^0$, in the local system of reference for $\alpha = 10^\circ$, $\alpha = 20^\circ$, $\alpha = 30^\circ$ are represented in Fig. 7., Fig. 8. and Fig. 9.

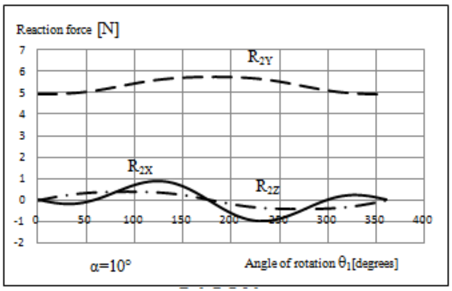

Fig. 7. The variation diagrams of reaction forces in the local reference system in the kinematic pair 2, for *α=10°*.

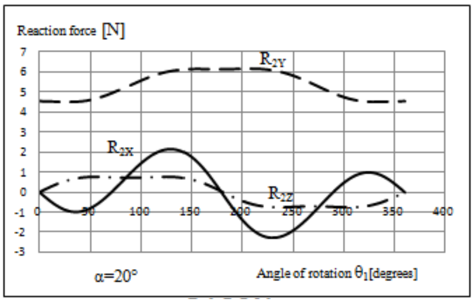

Fig. 8. The variation diagrams of reaction forces in the local reference system in the kinematic pair 2, for *α=20°*.

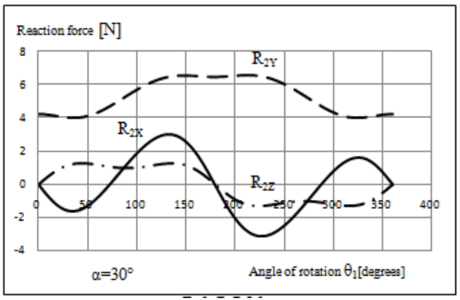

Fig. 9. The variation diagrams of reaction forces in the local reference system in the kinematic pair 2, for *α=30°*.

The maxim value of the reaction force R_{2Y} from kinematic pairs 2, increases from 5,3(N) ($\alpha = 0^\circ$) to 5,8(N) ($\alpha = 10^\circ$), to 6,1(N) ($\alpha = 20^\circ$) and to 6,3(N) ($\alpha = 30^\circ$).

V. CONCLUSIONS

The elastic calculation and the relative displacements method makes possible the determination of the reactions from the kinematic pairs of the mechanism. Following the numerical simulations made with the program realized using the calculation algorithm from the present paper, the following conclusions can be taken.

1) *For α=0°, in the general system of reference , the reactions force and moments after axes directions* OY_0 *and* OZ_0 *vary with the angle* θ_1*, and in the local system of reference they remain constant. The axial forces (after axe direction* OX_0*) from all kinematic pairs have the zero value .*
2) *For α≠0°, the reactions forces and moments aren't constant, both in the local and general system of reference [6]. Their variation depends on the angle* θ_1.
3) *The values of the reaction forces and moments from the kinematic pairs are increasing as the angle between cardan shafts α increases [6].*

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