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MATHEMATICAL MODEL FOR CALCULATING VIBRATION INHERENT FREQUENCIES AT BENDING FROM TWO-SHAFTS TRANSMISSION

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***Abstract*—**The average fibre of a elastic shaft as a horizontally resting on two simple supports, resiliently deforms under the action of own weight static, thus generating a mass eccentric to the axis of rotation of its own. The eccentric mass during the even rotation produces a centrifugal force, which increases the elastic deformation leading to the occurrence of bending vibration. The same happens in the case of shafts transmissions that can be treated with the elastic linkage systems of various sections, length and specific weights, elastic suspension media. This paper presents the mathematical model for calculating its own frequencies and vibration modes at bending for two-shafts transmission without technological deviations.

***Keywords*—**critical speeds, pulsation own, resonance, two-shafts transmission.

I. INTRODUCTION

SHAFTS transmission works optimal when the vibration inherent frequencies bending differs and are removed from harmonic frequencies disturbing, *2n, 3n*, (*n* the transmission speed), Fl. Dudita [1], Fl. Dudita, D. Diaconescu, Cr. Bohn, M. Neagoe, and R. Saulescu [2]. At these speeds the phenomenon of resonance, characterized by large amplitudes of movement in certain points or parts of the system, the vibration accompanied by demands and high stress or relative movement considerable, which can lead to fatigue fracture, malfunction, wear, vibration, so noise. The determination of these frequencies more precise (critical speeds) allows making constructive measures so as to avoid the undesirable effects of resonance.

According to literature, the calculation of critical speeds for two-shafts transmissions a relationship of the form, Fl. Dudita [1], Fl. Dudita, D. Diaconescu, Cr. Bohn, M. Neagoe, and R. Saulescu [2]

$$\mathbf{n = n_{crt} \approx 1{,}21 \cdot 10^7 \frac{\sqrt{D^2 + d^2}}{L^2}}. \qquad (1)$$

in which n_{crt} it is critical operating speed at which resonance occurs, *D* and *d*, an inner pipe outer diameter and *L* the length of the shaft connecting extreme cardan joints. This relationship, however, has a limited application, is valid only for the particular case of two-shafts transmission, with rigid liners and angles of small values, Fl. Dudita [1], Fl. Dudita, D. Diaconescu, Cr. Bohn, M. Neagoe, and R. Saulescu [2]. Based on the dynamic model with distributed mass, R. Voinea, D. Voiculescu and FL. Simion [3], A. Ripianu and I. Craciun [4], the two-shafts transmission, mathematical model is developed for calculating the vibration inherent frequencies bending will make an application number and will be draw conclusions.

II. GENERAL ASPECTS ON FIELD MATRICES AND VECTORS STATUS

Partial differential equation of the vibration of the free cross-bar to an average fibre length *l* and constant section *A*, leaning elastic, neglecting the effects of rotation and glissade the sections (see Fig. 1.), it is, R. Voinea, D. Voiculescu and FL. Simion [3], A. Ripianu and I. Craciun [4], N. Pandrea, S. Parlac and D. Popa [5]

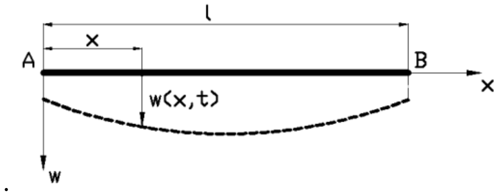

Fig. 1. Simple bar leaning

$$\frac{\partial^4 \mathbf{w}}{\partial \mathbf{x}^4} + \frac{\rho \mathbf{A}}{\mathbf{EI}} \frac{\partial^2 \mathbf{w}}{\partial \mathbf{t}^2} = \mathbf{0}. \qquad (2)$$

where: *w(x,t)* it is the arrow of the section bar *x* at the moment *t* to the equilibrium position, *ρ* the density, *E* modulus of elasticity, *I* geometric moment of inertia of the section bar in the main central axis normal to the plane *Axw*. A particular solution of the equation (2) it is

R. Voinea, D. Voiculescu and FL. Simion [3], A. Ripianu and I. Craciun [4]

$$\mathbf{w(x,t) = f(x)\cos(pt-\theta).} \tag{3}$$

where *f(x)* it is the amplitude of vibration harmonics, *p* pulsation own vibration and θ the rotational angle bar in section *x*, which meets conditions:

$$\frac{df}{dx} = \theta(x)\ ;\frac{d^2f}{dx^2} = -\frac{M(x)}{EI}\ ;\frac{d^3f}{dx^3} = -\frac{F(x)}{EI}. \quad (4$$

M, F the respective amplitudes of the bending moment and shear force acting in section *x* of bar. Are following notations, R. Voinea, D. Voiculescu and FL. Simion [3], A. Ripianu and I. Craciun [4]

1) $f_A\ ;\theta_A\ ;M_A\ ;F_A\ ;f_B\ ;\theta_B\ ;M_B\ ;F_B$ *obtained from equalities:*

$$\begin{aligned} f_A &= F(0) & f_B &= F(l) \\ \theta_A &= \theta(0) & \theta_B &= \theta(l) \\ M_A &= M(0) & M_B &= M(l) \\ F_A &= F(0) & F_B &= F(l). \end{aligned} \tag{5}$$

for amplitudes of displacements, rotations, bending moments and shear force of the end sections A and B results from the boundary conditions caused by the way the bar bearing.

2) Δ ,Δ_A, Δ_B *state vectors, defined by the relations:*

$$\begin{aligned} \Delta &= (f,\theta,M,F)^T \\ \Delta_A &= (f_A,\theta_A,M_A,F_A)^T \\ \Delta_B &= (f_B,\theta_B,M_B,F_B)^T. \end{aligned} \tag{6}$$

3) $f_j(z)$, *j=1,2,3,4 for Krâlov functions, defined by relations:*

$$\begin{aligned} f_1(z) &= \frac{ch(z)+\cos(z)}{2}\ ;f_2(z) = \frac{sh(z)+\sin(z)}{2} \\ f_3(z) &= \frac{ch(z)-\cos(z)}{2}\ ;f_4(z) = \frac{sh(z)-\sin(z)}{2}. \end{aligned} \tag{7}$$

where:

$$ch(z) = \frac{e^z+e^{-z}}{2}\ ;sh(z) = \frac{e^z-e^{-z}}{2}\ ;z = \alpha\cdot x\ . \tag{8}$$

4) *Parameter α defined by relations:*

$$\alpha = \sqrt[4]{p^2\frac{\rho A}{EI}}. \tag{9}$$

5) *F(z)– for matrix Krâlov, defined by relation:*

$$F(z) = \begin{pmatrix} f_1(z) & f_2(z) & f_3(z) & f_4(z) \\ f_4(z) & f_1(z) & f_2(z) & f_3(z) \\ f_3(z) & f_4(z) & f_1(z) & f_2(z) \\ f_2(z) & f_3(z) & f_4(z) & f_1(z) \end{pmatrix}. \tag{10}$$

6) α , α^{-1} *– for diagonal matrixes:*

$$\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\alpha} & 0 & 0 \\ 0 & 0 & -\frac{1}{\alpha^2 EI_w} & 0 \\ 0 & 0 & 0 & -\frac{1}{\alpha^3 EI_w} \end{pmatrix} \tag{11}$$

$$\alpha^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & -\alpha^2 EI_w & 0 \\ 0 & 0 & 0 & -\alpha^3 EI_w \end{pmatrix}.$$

7) *R - matrix of field, defined by relation* :

$$R = \alpha^{-1}\cdot F(\alpha x)\cdot\alpha. \tag{12}$$

With these relationships are obtained, I. Bulac [6], relations between state vectors:

$$\Delta = \alpha^{-1}\cdot F(\alpha x)\cdot\alpha\cdot\Delta_A\ ;\ \Delta_B = R\cdot\Delta_A. \tag{13}$$

If the bar is made up of sections of different lengths and sections (see Fig. 2.), then matrix of field sections are given by the relations:

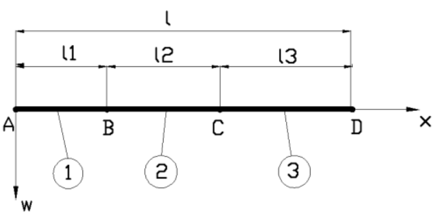

Fig. 2. Bar with sections of various lengths and sections.

$$R_i = \alpha_i^{-1}\cdot F(\alpha_i x_i)\cdot\alpha_i. \tag{14}$$

(i=1,2,3 case presented) and there are relations between state vectors:

$$\begin{aligned} \Delta_B &= R_1\cdot\Delta_A \\ \Delta_C &= R_2\cdot\Delta_B \\ \Delta_D &= R_3\cdot\Delta_C. \end{aligned} \tag{15}$$

from which resulting:

$$\Delta_D = R_3 \cdot R_2 \cdot R_1 \cdot \Delta_A. \qquad (16)$$

And matrix field for the entire bar is:

$$R = R_3 \cdot R_2 \cdot R_1. \qquad (17)$$

III. The Model with Distributed Weight for Two-Shafts Transmission

The dynamic model is carried out on a two-shafts transmission used in the SUV field whose construction model is shown in Fig. 3.a. Model constructive is attached to the equivalent dynamic model of Fig. 3.b., consists of three sections influences the shape of the shaft fork (l_1), pipe drive shaft (l_2) of the grooved portion of the drive shaft and a fork head connecting with the cross shaft of B (l_3). In sections A and D are located about the chassis elastic supports having elastic constants $k_A; k_D$.

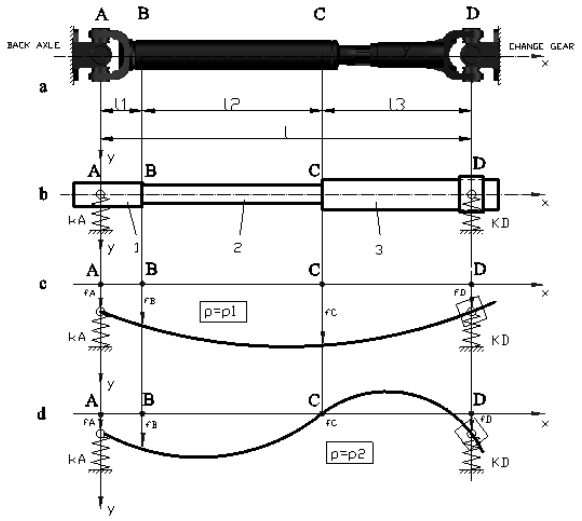

Fig. 3. The construction and mechanical equivalent model.

Assimilating the bonds from A and D with the articulations, one will obtain:

$$M_A = M_D = 0. \qquad (18)$$

shear forces of A and D are given by relations:

$$F_A = k_A f_A \; ; F_D = -k_D f_D. \qquad (19)$$

Considering the relation (6) of the state vectors are obtained:

$$\Delta_A = \left(\frac{F_A}{k_A}, \theta_A, 0, F_A \right)^T ; \Delta_D = \left(f_D, \theta_D, 0, -k_D f_D \right)^T. \qquad (20)$$

Using notations:

$$T_A = \begin{pmatrix} 0 & \frac{1}{k_A} \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$R^* = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \\ R_{31} & R_{32} \\ R_{41} & R_{42} \end{pmatrix} = R_3 \cdot R_2 \cdot R_1 \cdot T_A. \qquad (21)$$

state vectors from points A and D can be written as:

$$\Delta_A = T_A \cdot \begin{pmatrix} \theta_A \\ F_A \end{pmatrix}. \qquad (22)$$

and equality (16) convert:

$$\begin{pmatrix} f_D \\ \theta_D \\ 0 \\ -k_D f_D \end{pmatrix} = R^* \cdot \begin{pmatrix} \theta_A \\ F_A \end{pmatrix}. \qquad (23)$$

and hence the homogeneous system of equations is obtained in $\theta_A, F_A, \theta_D, f_D$:

$$\begin{cases} f_D = R_{11}\theta_A + R_{12}F_A \\ \theta_D = R_{21}\theta_D + R_{22}F_A \\ 0 = R_{31}\theta_A + R_{32}F_A \\ -k_D f_D = R_{41}\theta_A + R_{42}F_A \end{cases}. \qquad (24)$$

From the compatibility condition, D. Stanescu [7], above homogeneous linear system and their frequencies are determined. This compatibility condition is expressed by a transcendental equation, known as the characteristic equation. Describe their characteristic equation roots pulsations corresponding bending vibration mass distributed dynamic model adopted. For the system (24) to be non-trivial solution must be as determinant system is zero.

$$\Psi(p) = 0. \qquad (25)$$

where:

$$\Psi(p) = \begin{vmatrix} R_{11} & R_{12} & 1 \\ R_{21} & R_{22} & 1 \\ R_{31} & R_{32} & 0 \\ R_{41} & R_{42} & -k_D \end{vmatrix}. \qquad (26)$$

By solving the characteristic equation (25) set their pulses, p_1, p_2,…, corresponding dynamic model adopted for two-shafts transmission considered constructive model of which is shown in Fig. 3.

IV. THE REPRESENTATION VIBRATION INHERENT MODES BENDING

For the graphic representation of a mode of vibration it is necessary to calculate amplitudes (arrow) in different sections corresponding to the pulse of the vibration mode. Therefore, you can assign the arrow f_A the numerical value equal to the unit and then from the system (25) results:

$$\mathbf{F_A = k_A \; ; \theta_A = -\frac{R_{32}}{R_{31}} k_A}$$
$$\mathbf{f_D = k_A (R_{12} - R_{11}\frac{R_{32}}{R_{31}})} \qquad (27)$$
$$\mathbf{\theta_D = k_A (R_{22} - R_{21}\frac{R_{32}}{R_{31}}).}$$

Arrows f_B and f_C sections B and C are the first elements of the matrixes Δ_B , Δ_C data relationships:

$$\mathbf{\Delta_B = R_1 \cdot T_A \cdot \begin{pmatrix}\theta_A \\ F_A\end{pmatrix} \; ; \; \Delta_C = R_2 \cdot R_1 \cdot T_A \cdot \begin{pmatrix}\theta_A \\ F_A\end{pmatrix}}. \quad (28)$$

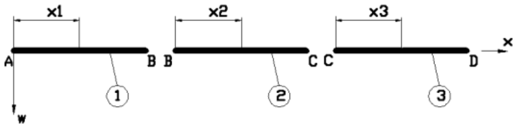

Fig. 4. Matrix calculation field for sections bars.

For way points (see Fig. 4.), the arrows are the first elements of the column matrixes Δ_1 ,Δ_2, Δ_3 calculated with the relations:

$$\mathbf{\Delta_1 = \alpha_1^{-1} \cdot F(\alpha_1 x_1) \cdot \alpha_1 \cdot T_A \cdot \begin{pmatrix}\theta_A \\ F_A\end{pmatrix}}$$
$$\mathbf{\Delta_2 = \alpha_2^{-1} \cdot F(\alpha_2 x_2) \cdot \alpha_2 \cdot \Delta_B} \qquad (29)$$
$$\mathbf{\Delta_3 = \alpha_3^{-1} \cdot F(\alpha_3 x_3) \cdot \alpha_3 \cdot \Delta_C.}$$

V. NUMERICAL APPLICATION

Consider a mobile two-shafts transmission equipping a SUV vehicle whose construction model is shown in Fig. 3., for the following construction features that are known and mechanical:

$$k_A = 85 \cdot 10^6 (N/m); k_D = 20 \cdot 10^6 (N/m); l_1 = 0{,}1(m)$$
$$l_2 = 1{,}0(m); l_3 = 0{,}4(m); A_1 = 19{,}6 \cdot 10^{-4}(m^2)$$
$$A_2 = 3{,}6 \cdot 10^{-4}(m^2); A_3 = 7{,}1 \cdot 10^{-4}(m^2)$$
$$\rho_1 = \rho_2 = \rho_3 = 7800(kg/m^3).$$

Based on an algorithm that I will present in future work and a computer program developed in Excel or obtained first and second pulsation own value p_1=323,5451(s^{-1}), p_2=1193,8478(s^{-1}). Corresponding to this pulse graphs were drawn at the bending vibration inherent modes shown in Fig. 5. and Fig. 6.

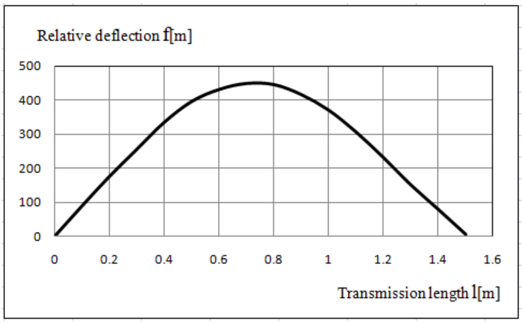

Fig. 5. Vibration mode corresponding to the first inherent pulse for real two-shafts transmission.

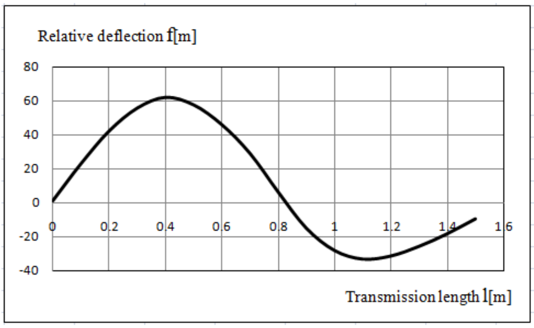

Fig. 6. Vibration mode corresponding to the second inherent pulse for real two-shafts transmission.

VI. CONCLUSIONS

The mathematical model presented together with the computer program algorithm and achieved makes it possible to study the inherent vibration frequencies and modes of two-shafts transmission bending. Based on numerical simulations can draw conclusions on the influence of mechanical and construction, the size pulsations and bending vibration modes.

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