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MATHEMATICAL MODEL FOR CALCULATING VIBRATION INHERENT FREQUENCIES AT BENDING FROM TWO-SHAFTS TRANSMISSION

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Abstract—The technological deviations of manufacturing and assembly with the unevenness material along the cardan shaft, the defects or the pronounced wear are disturbance factors for the functioning of cardan transmissions with influences on their own frequencies and vibration modes at bending. These deviations lead to the change of kinematic parameters and at the occurrence of further efforts in the kinematic pairs of the cardan joint mechanism. The variation of these forces being harmonic, makes it possible to study the influence of each type to technological deviation on their own frequencies and vibration modes at bending. In this paper It is presented the mathematical model for calculating vibration inherent frequencies at bending from two-shafts transmission with technological deviations.

Keywords—pulsation own, resonance, technological deviations, two-shafts transmission.

I. Introduction

THE supplementary efforts from the kinematic pairs are constant in the local systems of reference and harmonic range in the general system of reference, I. Bulac [1].

Harmonic ranging in general system of reference of origin is in center cardan cross, the axis OX_0 coincides with the rotation axis (see Fig. 1.), the forces and moments action as disturbing forces in the joints.

Fig. 1. The general system of reference of the two-shafts transmission

The component`s force on the OZ_0 direction axis is a permanent source of excitation, leading to the change of their own frequencies and vibration modes at bending.

Starting from the dynamic model with distributed mass, R. Voinea, D. Voiculescu and FL. Simion [2], A. Ripianu and I. Craciun [3], two-shafts transmission, the mathematical model is developed for calculating the vibration inherent frequencies bending, acording to the technological deviations of the component`s elements, and based on the results of numerical application conclusions will be drawn.

II. Technological Deviations in Plücker Coordinates

The 4r Symmetrical Spherical Quadrilateral Mechanism, from which derived the cardan joint mechanism, that cinematically speaking describes the movement of a cardan mechanism, is of third family and, Fl. Dudita [4], Fl. Dudita, D. Diaconescu, Cr. Bohn, M. Neagoe, and R. Saulescu [5], N. Dumitru, Gh. Nanu, and Daniela Vintilă [6], N. Pandrea and D. Popa [7], it is multiple statically undetermined. For determining the reactions from the rotation kinematic pairs A, B, C, D it is necessary to use a linearly calculus, N. Pandrea [8], and the mathematical model that is obtained is applied to the efforts caused, I. Bulac [1], by technological deviations.

In the elastically linear calculation it is considered that the kinematic pair from A is fixed and the technological deviation of the AB element, (see Fig. 1.), is given in the local reference system $Bxyz$, by the small rotation angle $\bar{\theta}_B^l$ and by the small displacement $\bar{\delta}_B^l = B\bar{B}'$.

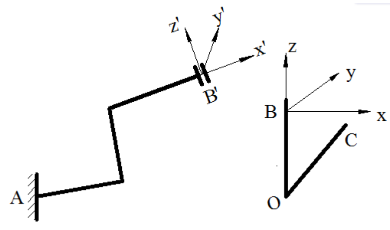

Fig. 2. The technological deviation of the element AB

In plücker coordinates, I. Bulac [1], N. Pandrea, [8], the deviations Δ_B^l, in the $Bxyz$ local reference system is written under the form:

$$\Delta_B^l = \left(\theta_{Bx}^l, \theta_{By}^l, \theta_{Bz}^l, \delta_{Bx}^l, \delta_{By}^l, \delta_{Bz}^l\right)^T. \qquad (1)$$

where $\left(\theta_{Bx}^l, \theta_{By}^l, \theta_{Bz}^l, \delta_{Bx}^l, \delta_{By}^l, \delta_{Bz}^l\right)^T$ are the projections on the local axis of the vectors $\bar{\theta}_B^l, \bar{\delta}_B^l$.

The numerical simulations made, I. Bulac [1] for the 4r symmetrical spherical quadrilateral mechanism, for the steel parts of the same length of 0,04(m) and the same 0,02(m) diameter, with a local deviation from perpendicularly of 0.0001 (m) in the plane *Oxy* (the local system of reference of the point B) at the spindle of the cardan cross from joint B, led to a variation force from A, of the form

$$R_{AZ} = 100\cos(\theta_1 - \frac{\pi}{2}) \qquad (2)$$

where R_{AZ} is the reaction from A on the direction of the axis OZ_0 and θ_1 is the rotation angle of the element AB. In the case of uniform rotation with the angularly speed ω the angle θ_1 is given by the relation

$$\theta_1 = \omega t \qquad (3)$$

and the excitation produced by only one of the deviations is harmonic.

III. THE DISTRIBUTED MASS DYNAMIC MODEL OF THE TWO- SHAFTS TRANSMISSION

To the constructive solution of the two-shafts transmission, (see Fig. 3.a.), is assigned the mathematical model (see Fig. 3.b.),.

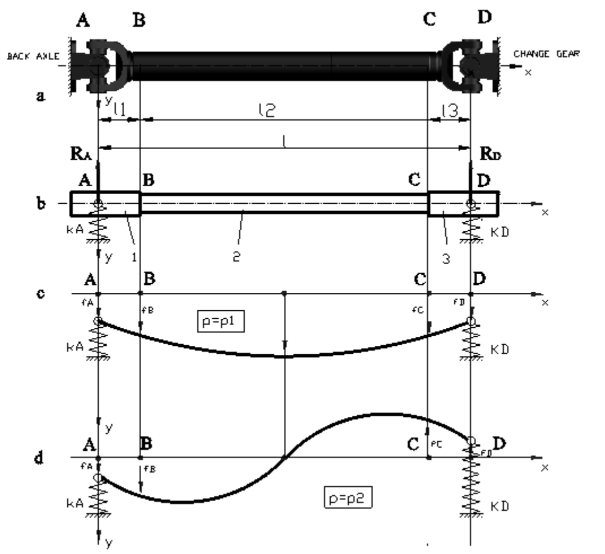

Fig. 3. Constructive model and equivalent mechanical model.

in the sections D, being situated with the elastic bearing that has the constants k_A, k_D and the harmonic exciting forces R_A, R_D with the amplitudes $\tilde{R}_A, \tilde{R}_D$, activating in the sections A, D, which in the new system chosen acts on the direction axis Oy.

Next the notations are used, R. Voinea, D. Voiculescu and FL. Simion [2], A. Ripianu and I. Craciun [3]

1) f_i, θ_i, M_i, F_i - *as the deflection, rotation, deflection moment and the cutting force from the current section*
2) Δ_i ; Δ_A ; Δ_B ; Δ_C ; Δ_D - *state vectors, defined by the relations:*

$$\begin{aligned}
\Delta_i &= (f_i, \theta_i, M_i, F_i)^T \\
\Delta_A &= (f_A, \theta_A, M_A, F_A)^T \\
\Delta_B &= (f_B, \theta_B, M_B, F_B)^T \\
\Delta_C &= (f_C, \theta_C, M_C, F_C)^T \\
\Delta_D &= (f_D, \theta_D, M_D, F_D)^T.
\end{aligned} \qquad (4)$$

3) $x_i, \rho_i, A_i, E_i, I_{yi}$ - *respectively, the length, density, area, transverse modulus of elasticity and geometrical moment of inertia mainly for the section corresponding to the index* $i = 1,2,3$
4) *The parameters* $\alpha_i ; z_i$ *defined by the relations*:

$$\alpha_i = \sqrt[4]{p^2 \frac{\rho_i A_i}{E_i Iyi}}; z_i = \alpha_i \cdot x_i. \qquad (5)$$

where p is the vibration inherent pulse.

1) chz, shz - *sin and cos the hyperbolic functions*

$$\begin{aligned}
ch(z_i) &= \frac{e^{z_i} + e^{-z_i}}{2} \\
sh(z_i) &= \frac{e^{z_i} - e^{-z_i}}{2} \ .
\end{aligned} \qquad (6)$$

5) $f_j(z_i)$, *j=1,2,3,4 the Krâlov functions defined by relations:*

$$\begin{aligned}
f_1(z_i) &= \frac{ch(z_i) + cos(z_i)}{2} \\
f_2(z_i) &= \frac{sh(z_i) + sin(z_i)}{2} \\
f_3(z_i) &= \frac{ch(z_i) - cos(z_i)}{2} \\
f_4(z_i) &= \frac{sh(z_i) - sin(z_i)}{2}.
\end{aligned} \qquad (7)$$

6) $F(z_i)$ – *the Krâlov matrixes defined by relations:*

$$\mathbf{F(z_i)}=\begin{pmatrix} f_1(z_i) & f_2(z_i) & f_3(z_i) & f_4(z_i) \\ f_4(z_i) & f_1(z_i) & f_2(z_i) & f_3(z_i) \\ f_3(z_i) & f_4(z_i) & f_1(z_i) & f_2(z_i) \\ f_2(z_i) & f_3(z_i) & f_4(z_i) & f_1(z_i) \end{pmatrix}. \quad (8)$$

7) α , α^{-1} *– for diagonal matrixes:*

$$\alpha=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\alpha} & 0 & 0 \\ 0 & 0 & -\frac{1}{\alpha^2 EI_w} & 0 \\ 0 & 0 & 0 & -\frac{1}{\alpha^3 EI_w} \end{pmatrix} \quad (9)$$

$$\alpha^{-1}=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & -\alpha^2 EI_w & 0 \\ 0 & 0 & 0 & -\alpha^3 EI_w \end{pmatrix}.$$

8) $R_i, i-1,2,3$, *the field matrixes of sections, I. Bulac [9]:*

$$\mathbf{R_i}=\alpha_i^{-1}\cdot \mathbf{F}(\alpha_i x_i)\cdot\alpha_i. \quad (10)$$

9) R, *the field matrix for all two-shafts transmission limited of the points A, D:*

$$\mathbf{R}=\mathbf{R_3}\cdot\mathbf{R_2}\cdot\mathbf{R_1}. \quad (11)$$

With these notations, in the absence of the disturbing forces, is obtained the equality:

$$\Delta_D=\mathbf{R}\cdot\Delta_A. \quad (12)$$

and in the presence of disturbing forces is obtained:

$$\Delta_D+\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\tilde{R}_D \end{pmatrix}=\mathbf{R}\cdot\Delta_A+\mathbf{R}\cdot\begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_A \end{pmatrix}. \quad (13)$$

By complying the bonds from sections A and D with the articulations results that:

$$\begin{aligned} M_A &= M_D = 0 \\ F_A &= k_A f_A \\ F_D &= -k_D f_D \end{aligned} \quad (14)$$

and then the equality (13) in the condition of harmonic excitations becomes:

$$\begin{pmatrix} \tilde{f}_D \\ \tilde{\theta}_D \\ 0 \\ -k_D\tilde{f}_D \end{pmatrix}-\mathbf{R}\cdot\begin{pmatrix} \tilde{F}_A/k_A \\ \tilde{\theta}_A \\ 0 \\ \tilde{F}_A \end{pmatrix}=\mathbf{R}\cdot\begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_A \end{pmatrix}+\begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_D \end{pmatrix}. \quad (15)$$

where by $\tilde{f}_D, \tilde{\theta}_D, \tilde{\theta}_A, \tilde{f}_A$ noted the amplitudes of sizes $f_D, \theta_D, \theta_A, f_A$.

Next, using the notations:

$$\mathbf{R^*}=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -k_D & 0 & 0 & 0 \end{pmatrix}-\mathbf{R}\cdot\begin{pmatrix} 0 & 0 & 0 & 1/k_A \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (16)$$

and

$$\mathbf{F^*}=\mathbf{R}\cdot\begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_A \end{pmatrix}+\begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_D \end{pmatrix}. \quad (17)$$

from (15) is obtained the equality

$$\mathbf{R^*}\cdot\begin{pmatrix} \tilde{f}_D \\ \tilde{\theta}_D \\ \tilde{\theta}_A \\ \tilde{F}_A \end{pmatrix}=\mathbf{F^*}. \quad (18)$$

from which results

$$\begin{pmatrix} \tilde{f}_D \\ \tilde{\theta}_D \\ \tilde{\theta}_A \\ \tilde{F}_A \end{pmatrix}=\mathbf{R^{*-1}}\mathbf{F^*}. \quad (19)$$

and hence the homogeneous system of equations is obtained in $\tilde{\theta}_A; \tilde{F}_A; \tilde{\theta}_D; \tilde{f}_D$.

The state vectors sizes amplitudes from sections B, C are given by the relations:

$$\begin{pmatrix} \tilde{f}_B \\ \tilde{\theta}_B \\ \tilde{M}_B \\ \tilde{F}_B \end{pmatrix}=\mathbf{R_1}\cdot\begin{pmatrix} \tilde{F}_A/k \\ \tilde{\theta}_A \\ 0 \\ \tilde{F}_A \end{pmatrix}+\begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_A \end{pmatrix}. \quad (20)$$

for the amplitudes from the section B

$$\begin{pmatrix} \tilde{f}_C \\ \tilde{\theta}_C \\ \tilde{M}_C \\ \tilde{F}_C \end{pmatrix} = \mathbf{R}_2 \cdot \begin{pmatrix} \tilde{f}_B \\ \tilde{\theta}_B \\ \tilde{M}_B \\ \tilde{F}_B \end{pmatrix}. \qquad (21)$$

for the amplitudes from the section C.

IV. NUMERICAL APPLICATION

It is considered a two-shafts transmission of an off-road vehicle whose construction model is shown in Fig. 3., for the following construction features that are known and mechanical:

$$k_A = 85 \cdot 10^6 (N/m);\ k_D = 20 \cdot 10^6 (N/m);\ l_1 = l_3 = 0{,}1(m);$$

$$l_2 = 0{,}8(m);\ A_1 = A_3 = 19{,}6 \cdot 10^{-4}(m^2);\ A_2 = 3{,}6 \cdot 10^{-4}(m^2);$$

$$\rho_1 = \rho_2 = \rho_3 = 7800(kg/m^3)$$

with a local deviation from perpendicularly of 0.0001 (m) at one of the spindle of the cardan cross of the cardam joint mechanism from the transmission of the component.

Based on an algorithm that I will be present in future work and a computer program developed in Excel or obtained for first and second pulsation own value p_1=749,366(s^{-1}), p_2=2590,4921(s^{-1}).

Corresponding to this pulse, graphs were drawn at the bending vibration inherent modes shown in Fig. 4. and Fig. 5.

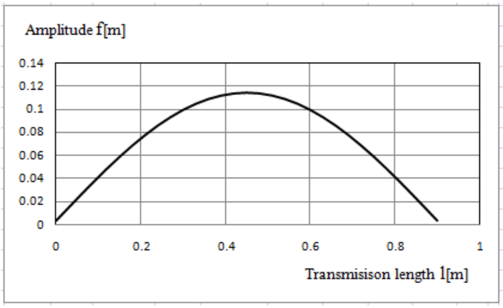

Fig. 4. The first way of vibration, corresponding to the first vibration inherent frequencies at bending.

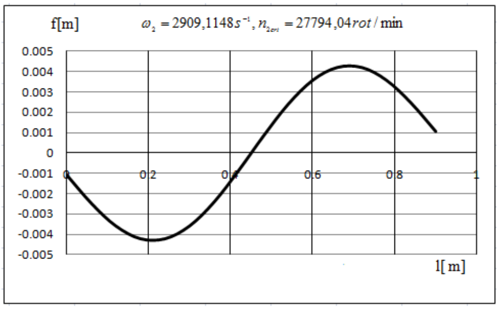

Fig. 5. The second way of vibration, corresponding to the second vibration inherent frequencies at bending.

V. CONCLUSIONS

Based on the mathematical model presented, of the algorithm and of the program developed can be determinated the influence of each type of technological deviation an their own frequencies and vibration modes at bending of two-shafts transmissions.

The results of the numerical simulations may be used to determine these deviations thus inherent frequencies obtained not to overlap over the frequencies of the disruptive forces which are generally known.

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