Mathematical model for the calculation of vibration inherent frequencies at bending from three-shafts transmission

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Abstract. The three-shafts transmissions can be considered as being elastic systems of joint bars of different lengths, sections, and specific weights. During functioning (spinning), due to its own weight or to some technological deviations of execution and assembling will appear vibrations maintained by the bending. These maintained vibrations frequently lead to get the resonance regim, when oscillations amplitude grow leading to the deterioration or destruction of the cardanic transmission. This paper presents an mathematical model for calculating the pulsation and vibration modes at bending from three-shafts transmission.

1 Introduction

For the good functioning of the cardanic transmission it is required to avoid the vicinity of the own frequencies to the frequency of the disturbing sources and taking some constructive measures of damping of the vibrations so that the effects of the amplitudes of resonance not to overtake the accepted limits, Fl. Dudita [1], Fl. Dudita, D. Diaconescu, Cr. Bohn, M. Neagoe, and R. Saulescu [2]. Knowing exactly the connection relationships between the mechanical and constructive parameters of the cardanic transmissions and frequencies and vibration modes at bending, allows taking some proper technical measures so that the possible resonance effects to be avoided or limited as possible as it can be. As in the case of the tow-shafts transmission, for achieve the mathematical model for calculating vibration inherent frequencies at bending, is starting from the dynamic model with distributed mass, Fl. Dudita, D. Diaconescu, Cr. Bohn, M. Neagoe, and R. Saulescu [2], R. Voinea, D. Voiculescu and FL. Simion [3], A. Ripianu and I. Craciun [4].

2 General aspects on field matrixex and vectors status

In Fig. 1, is given the constructive solution of the three-shafts transmission, used in the SUV field, whose.

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Fig. 1. The constructive model of the three-shafts transmission

The constructive solution presented in Fig. 1., is attached to the equivalent mechanical model presented in Fig. 2. consists of two bars, AD and DH, elastically articulated in D, that have the elastically constants k_A, k_E, k_H in the sections A , E and H.

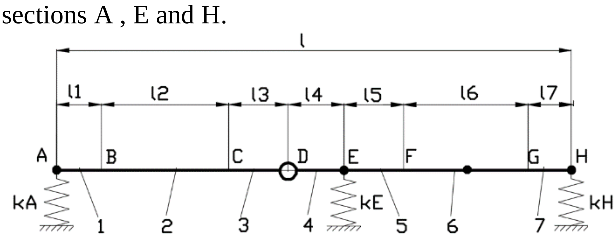

Fig. 2. The equivalent mechanical model

The dynamic modeling with distributed masses of the three-shafts transmission and the first two vibration modes at bending is presented in Fig. 3.

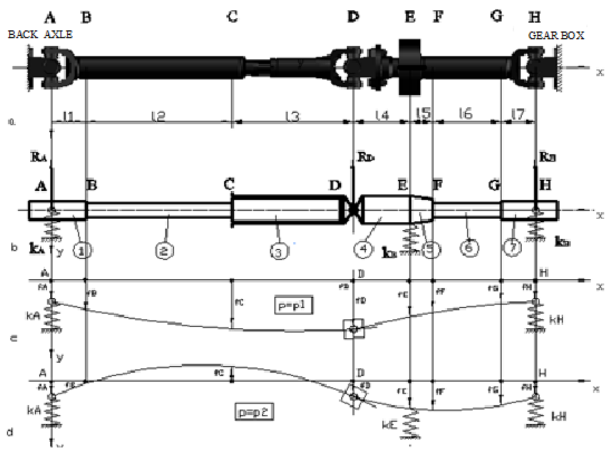

Fig. 3. The dynamic model with distributed masses attached of the three-shafts transmission

Next the notations are used, Fl. Dudita, D. Diaconescu, Cr. Bohn, M. Neagoe, and R. Saulescu [2], A. Ripianu and

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I. Craciun [4], R. Voinea, D. Voiculescu and FL. Simion [5], N. Pandrea, S. Parlac and D. Popa [6]

1) f_i, θ_i, M_i, F_i as the deflection, rotation, deflection moment and the cutting force from the current section;

2) The state vectors: Δ_i ; Δ_A ; Δ_B ; Δ_C ; Δ_D ; Δ_E ; Δ_F ; Δ_G ; Δ_H defined by the relations:

$$\begin{aligned} \Delta_i &= (f_i, \theta_i, M_i, F_i)^T \\ \Delta_A &= (f_A, \theta_A, M_A, F_A)^T \\ \Delta_B &= (f_B, \theta_B, M_B, F_B)^T \\ \Delta_C &= (f_C, \theta_C, M_C, F_C)^T; \\ \Delta_D &= (f_D, \theta_D, M_D, F_D)^T. \\ \Delta_E &= (f_E, \theta_E, M_E, F_E)^T; \\ \Delta_F &= (f_F, \theta_F, M_F, F_F)^T \\ \Delta_G &= (f_G, \theta_G, M_G, F_G)^T; \\ \Delta_H &= (f_H, \theta_H, M_H, F_H)^T. \end{aligned} \quad (1)$$

3) $x_i, \rho_i, A_i, E_i, I_{yi}$ - respectively, the length, density, area, transverse modulus of elasticity and geometrical moment of inertia mainly for the section corresponding to the index $i = 1,2,3,...7$

4) The parameters $\alpha_i ; z_i$ defined by the relations:

$$\alpha_i = \sqrt[4]{p^2 \frac{\rho_i A_i}{E_i Iyi}}; z_i = \alpha_i \cdot x_i. \quad (2)$$

where p is the vibration inherent pulse.

5) chz, shz - sin and cos the hyperbolic functions

$$ch(z_i) = \frac{e^{z_i} + e^{-z_i}}{2} ; \quad sh(z_i) = \frac{e^{z_i} - e^{-z_i}}{2} . \quad (3)$$

6) $f_j(z_i)$, j=1,2,3,4 the Krâlov functions defined by relations:

$$\begin{aligned} f_1(z_i) &= \frac{ch(z_i) + cos(z_i)}{2}; \\ f_2(z_i) &= \frac{sh(z_i) + sin(z_i)}{2} \\ f_3(z_i) &= \frac{ch(z_i) - cos(z_i)}{2}; \\ f_4(z_i) &= \frac{sh(z_i) - sin(z_i)}{2}. \end{aligned} \quad (4)$$

7) $F(z_i)$ – the Krâlov matrixes defined by relations:

$$F(z_i) = \begin{pmatrix} f_1(z_i) & f_2(z_i) & f_3(z_i) & f_4(z_i) \\ f_4(z_i) & f_1(z_i) & f_2(z_i) & f_3(z_i) \\ f_3(z_i) & f_4(z_i) & f_1(z_i) & f_2(z_i) \\ f_2(z_i) & f_3(z_i) & f_4(z_i) & f_1(z_i) \end{pmatrix}. \quad (5)$$

8) α , α^{-1} – for diagonal matrixes:

$$\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\alpha} & 0 & 0 \\ 0 & 0 & -\frac{1}{\alpha^2 EI_w} & 0 \\ 0 & 0 & 0 & -\frac{1}{\alpha^3 EI_w} \end{pmatrix}; \quad \alpha^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & -\alpha^2 EI_w & 0 \\ 0 & 0 & 0 & -\alpha^3 EI_w \end{pmatrix}. \quad (6)$$

9) $R_i, i - 1,2,3,...7$, the field matrixes of sections

$$R_i = \alpha_i^{-1} \cdot F(\alpha_i x_i) \cdot \alpha_i. \quad (7)$$

3 Determining vibration inherent frequencies at bending

By assimilating the bearings from sections A, D, H with the joints, results that

$$M_A = M_D = M_H = 0. \quad (8)$$

and by also taking into account the elastic brackets results

$$\begin{aligned} F_A &= k_A f_A \\ F_E^d &= F_E^s + k_E f_E \\ F_H &= -k_H f_H. \end{aligned} \quad (9)$$

where with F_E^s is noted the cutting force from the left side of the section from E and with F_E^d the cutting force from the right side of the E section.In the same was with θ_D^s is the revolution of the section from D of the AD bar and with θ_D^d the revolution of the section from D of the DH bar.

Using notations:

$$T_A = \begin{pmatrix} 0 & \frac{1}{k_A} \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad (10)$$

for the AD bar, the status vectors in the final sections are given by the relation:

$$\begin{pmatrix} \mathbf{f_D} \\ \theta_\mathbf{D} \\ \mathbf{0} \\ -\mathbf{k_D f_D} \end{pmatrix} = \mathbf{R}^* \cdot \begin{pmatrix} \theta_\mathbf{A} \\ \mathbf{F_A} \end{pmatrix}. \tag{11}$$

where R^* are given by the relation:

$$\mathbf{R}^* = \begin{pmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{21} & \mathbf{R}_{22} \\ \mathbf{R}_{31} & \mathbf{R}_{32} \\ \mathbf{R}_{41} & \mathbf{R}_{42} \end{pmatrix} = \mathbf{R}_3 \cdot \mathbf{R}_2 \cdot \mathbf{R}_1 \cdot \mathbf{T_A}. \tag{12}$$

From the matrix equality (11) the following equalities are obtained:

$$\begin{cases} \mathbf{f_D} = \mathbf{R}_{11}\theta_\mathbf{A} + \mathbf{R}_{12}\mathbf{F_A} \\ \theta_\mathbf{D} = \mathbf{R}_{21}\theta_\mathbf{D} + \mathbf{R}_{22}\mathbf{F_A} \\ \mathbf{0} = \mathbf{R}_{31}\theta_\mathbf{A} + \mathbf{R}_{32}\mathbf{F_A} \\ -\mathbf{k_D f_D} = \mathbf{R}_{41}\theta_\mathbf{A} + \mathbf{R}_{42}\mathbf{F_A} \end{cases}. \tag{13}$$

from which, with the notations :

$$\lambda_1 = \mathbf{R}_{12} - \mathbf{R}_{11}\frac{\mathbf{R}_{32}}{\mathbf{R}_{31}}$$
$$\lambda_2 = \mathbf{R}_{42} - \mathbf{R}_{41}\frac{\mathbf{R}_{32}}{\mathbf{R}_{31}}. \tag{14}$$

results :

$$\theta_\mathbf{A} = -\frac{\mathbf{R}_{32}}{\mathbf{R}_{31}}\mathbf{F_A}; \mathbf{f_D} = \lambda_1\mathbf{F}; \mathbf{F_D} = \lambda_2\mathbf{F}. \tag{15}$$

In the same conditions the matrix:

$$\Delta_\mathbf{D}^\mathbf{d} = \left(\mathbf{f_D}, \theta_\mathbf{D}^\mathbf{d}, \mathbf{0}, \mathbf{F_D}\right)^\mathbf{T}. \tag{16}$$

with the notation :

$$\mathbf{T_D} = \begin{pmatrix} \mathbf{0} & \lambda_1 \\ \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \lambda_2 \end{pmatrix}. \tag{17}$$

is written under:

$$\Delta_\mathbf{D}^\mathbf{d} = \mathbf{T_D} \cdot \left(\theta_\mathbf{D}^\mathbf{d}, \mathbf{F_A}\right)^\mathbf{T}. \tag{18}$$

The column matrix:

$$\Delta_\mathbf{E}^\mathbf{s} = \left(\mathbf{f_E}, \theta_\mathbf{E}, \mathbf{M_E}, \mathbf{F_E^s}\right)^\mathbf{T}. \tag{19}$$

is obtained from the equality:

$$\Delta_\mathbf{E}^\mathbf{s} = \mathbf{P} \cdot \left(\theta_\mathbf{D}^\mathbf{d}, \mathbf{F_A}\right)^\mathbf{T}. \tag{20}$$

where :

$$\mathbf{P} = \mathbf{R}_4 \cdot \mathbf{T_D}. \tag{21}$$

From (20) are obtained:

$$\mathbf{f_E} = \mathbf{P}_{11}\theta_\mathbf{D}^\mathbf{d} + \mathbf{P}_{12}\mathbf{F_A}$$
$$\mathbf{F_E^s} = \mathbf{P}_{41}\theta_\mathbf{D}^\mathbf{d} + \mathbf{P}_{42}\mathbf{F}. \tag{22}$$

and taking into account the second relation (9) it results:

$$\mathbf{F_E^d} = \mathbf{P}_{41}\theta_\mathbf{D}^\mathbf{d} + \mathbf{P}_{42}\mathbf{F_A} + \mathbf{k_E}(\mathbf{P}_{11}\theta_\mathbf{D}^\mathbf{d} + \mathbf{P}_{12}\mathbf{F_A}). \tag{23}$$

In these conditions:

$$\Delta_\mathbf{E}^\mathbf{d} = \left(\mathbf{f_E}, \theta_\mathbf{E}, \mathbf{M_E}, \mathbf{F_E^d}\right)^\mathbf{T}. \tag{24}$$

with the notation:

$$\tilde{\mathbf{P}} = \mathbf{P} + \mathbf{k_E} \cdot \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{P}_{11} & \mathbf{P}_{12} \end{pmatrix}. \tag{25}$$

is written:

$$\Delta_\mathbf{E}^\mathbf{d} = \tilde{\mathbf{P}} \cdot \left(\theta_\mathbf{D}^\mathbf{d}, \mathbf{F_A}\right)^\mathbf{T}. \tag{26}$$

and the matric equality:

$$\mathbf{Q} = \mathbf{R}_7 \cdot \mathbf{R}_6 \cdot \mathbf{R}_5 \cdot \Delta_\mathbf{E}^\mathbf{d}. \tag{27}$$

with the notation

$$\Delta_\mathbf{H} = \mathbf{R}_7 \cdot \mathbf{R}_6 \cdot \mathbf{R}_5 \cdot \tilde{\mathbf{P}}. \tag{28}$$

becomes:

$$\begin{pmatrix} \mathbf{f_H} \\ \theta_\mathbf{H} \\ \mathbf{0} \\ -\mathbf{k_H f_H} \end{pmatrix} = \mathbf{Q} \cdot \begin{pmatrix} \theta_\mathbf{D}^\mathbf{d} \\ \mathbf{F_A} \end{pmatrix}. \tag{29}$$

From here is obtained the homogenous equation system in θ_D^d, F_A and f_H

$$\begin{cases} f_H = Q_{11}\theta_D^d + Q_{12}F_A \\ \theta_D = Q_{21}\theta_D^d + Q_{22}F_A \\ 0 = Q_{31}\theta_D^d + Q_{32}F_A \\ -k_H f_H = Q_{41}\theta_D^d + Q_{42}F_A \end{cases}. \quad (30)$$

that admits a solution different then zero if:

$$\begin{pmatrix} Q_{11} & Q_{12} & 1 \\ Q_{31} & Q_{32} & 0 \\ Q_{41} & Q_{42} & -k_H \end{pmatrix} = 0. \quad (31)$$

and also if the pulsation p is verified by:

$$\Psi_p = Q_{31}Q_{42} - Q_{41}Q_{32} - k_H(Q_{11}Q_{32} - Q_{12}Q_{31}) = 0. \quad (32)$$

By solving the characteristic equation (32) set their pulses $p_1, p_2, \ldots,$.

4 Numerical application

Consider a mobile three-shafts transmission equipping a SUV vehicle whose construction model is shown in Fig. 3 for the following construction features that are known and mechanical:

$$k_A = k_H = 85 \cdot 10^6 (N/m); k_E = 2 \cdot 10^6 (N/m);$$
$$l_1 = 0{,}07(m); l_2 = 0{,}59(m); l_3 = 0{,}25(m);$$
$$l_4 = 0{,}12(m); l_5 = 0{,}05(m); l_6 = 0{,}36(m);$$
$$l_7 = 0{,}07(m);$$
$$A_1 = A_3 = A_4 = A_5 = A_7 = 19{,}6 \cdot 10^{-4}(m^2);$$
$$A_2 = A_6 = 4 \cdot 10^{-4}(m^2)$$
$$\rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho_5 = \rho_6 = \rho_7 = 7800(kg/m^3).$$

Based on an algorithm that I will present in future work and a computer program developed in Excel or obtained first and second pulsation own value $p_1=155(s^{-1})$, $p_2=1072(s^{-1})$. Corresponding to this pulse graphs were drawn at the bending vibration inherent modes shown in Fig. 4 and Fig. 5.

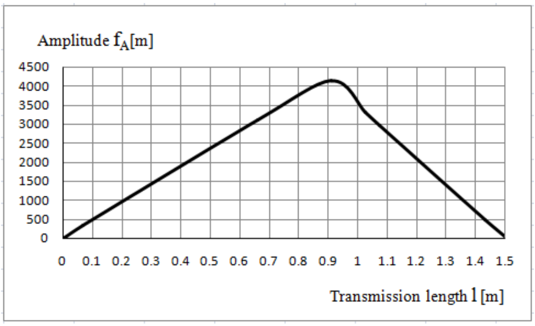

Fig. 4. Vibration mode corresponding to the first inherent pulse for real three-shafts transmission.

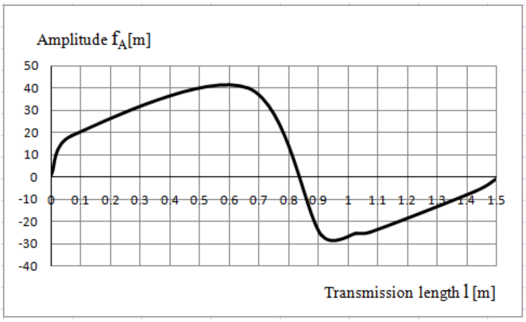

Fig. 5. Vibration mode corresponding to the second inherent pulse for real three-shafts transmission.

5 Conclusions

The mathematical model presented the algorithm and the program developed, can be determinated the influence of each mechanical and constructive parameter on own frequencies and vibration modes at bending of three-shafts transmissions. Also taking, into account the calculation program, through this method can be determinate the influence of the elastically constants from points A, D and H over the critical speeds.

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